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Bård Harstad

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Besøksadresse: Grønlandsleiret 25
Addresse: Postboks 8159 Dep.
0033 Oslo
Internett: www.nupi.no
E-post: pub@nupi.no Fax: [+47] 22177015 Tel: [+47] 22056500

# Integration and Regionalization - a political economic analysis 

Bård Harstad *

Institute for international economic studies<br>Stockholm University<br>SE-106 91 Stockholm<br>and<br>Norwegian Institute of International Affairs<br>Grønlandsleiret 25<br>P.O. Box 8159 Dep., o033 Oslo<br>September 2000

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[Abstract] This paper examines the incentives for political integration in a situation with a non-excludable public good. The model emphasizes inter-regional differences in sizes and preferences for the public good. In such a two-country model, Ellingsen $(1998)^{1}$ characterizes the cases in which integration is an equilibrium. This paper includes a third region, and finds that a whole range of interesting and observable issues arise, which the two-country model is unable to capture. Depending on the relative differences in sizes and preferences among regions, the integration problem may be described as a prisoner's dilemma, a coordination game or as a hawk-dove game. Multiple equilibria may exist as well as equilibria with no integration; partly integration; conditionally integration and exclusion from the coalition. The extreme case where the public good is global (beneficial to all) is discussed, as well as an extreme where it is local (beneficial only to the closest neighbors).

Key words: Jurisdictions; Public goods; Free riding; Integration; Exclusion dlj aflj
JEL classification: D7, H4, H7
${ }^{1}$ Ellingsen, T. (1998): "Externalities vs internalities: a model of political integration", Journal of Public Economics (68).

## 1. Introduction

There are a huge number of different forces in work when regions integrate or separate with each other. There are centripetal forces making regions integrate, as well as centrifugal forces separating jurisdictions. By analyzing the different forces we not only understand and predict integration and separation better, but it will also help to design the optimal decision level in a hierarchically built government.

One example of conflict between centripetal and centrifugal forces is described in Bolton and Roland (1997). Here risk-averse individuals benefit from a convex tax-schedule when the income is stochastic. This insurance is better the larger the jurisdiction, but it has a negative expected value for high-income regions.

Another kind of conflict is described by Cremer and Palfrey (1996): An individual knows only her own preference for some political variable, but she also knows that her preference is more correlated to other preferences in her own region than in other regions. On the other hand, the political variable in integration is more predictable.

A third example of conflicting forces is described in Alesina and Spolaore (1997). They consider the trade-off between economies of scale (which favors integration) and regional heterogeneity (which favors separation). Closely linked to this is Ellingsen (1998): To avoid externalities a large jurisdiction is preferable when there is a public good. On the other hand, differences in preferences arise internalities because the majority neglects the minority. In addition, it might be beneficial for a region to separate because it may then free ride. In many cases this sounds like a plausible description of reality. There are in fact a number of positive and negative externalities between countries. Coordinating public goods as defense, financial policy or pollution controls are reasonable benefits when regions integrate. It is also plausible that the preferences over these tasks vary across regions. The analysis of European Integration by Inman and Rubinfeld (1992) is a typical example of this argument being applied to a practical case.

As functions of the relative differences in size and preferences among regions, Ellingsen (1998) characterizes the optimal and the equilibrium political regime in a model for two regions. Among his results is that integration is more likely if the regions are of equal size or if preferences for the public good is strong. In separation only one region A is producing the public good.

This paper makes some natural extensions of this model. A third region C is included in the analysis and surprisingly many new issues arise. Moreover, the case where the public good is global (equal beneficial for all regions) is analyzed, as well as the other extreme where it is local (beneficial only close to the production location).

The first case is discussed in section three, after presenting the model and some of Ellingsen's results in section two. When the producing region A is large and has strong preferences for the public good, no other region benefits from integrating with A . If A is less dominating, the other two regions B and C might benefit if both are integrating with A , because then they can afford a much larger production of the public good. In that case, B and C's decision whether to integrate or not may be described as a prisoner's dilemma game. For other differences in sizes and preferences, a coordination game or a
hawk-dove game better captures the essence in the problem. A, for itself, might prefer to integrate with only one if B and C together are in majority in a full integration. The circumstances under which these and other different situations appear are discussed.

Section four assumes that the regions are laying more dispersed, such that the production is beneficial for the region itself and its closest neighbors only. In integration it is then important whether the production shares are tradable (allowing one region to pay for the production in another region) or not. In separation, two equilibria may be possible: one where only the region in "the middle" is producing, and another where the two regions not in the middle are producing. Both cases are discussed, and it is explained why the incentives to integrate can either be stronger or weaker compared to section three. Moreover, the region in the middle may prefer to exclude other to make or join a coalition.

## 2. The model

This section presents a simplified version of Ellingsen (1998), ${ }^{3}$ generalized to an arbitrary number of regions. The main idea is that the presence of a public good makes large jurisdictions preferable to avoid externalities. On the other hand, inter-regional differences in the preference for the public good create internalities because the majority neglects the minority. In addition, separation may be preferable for one region because it can then free ride.

## General notation

There are n individuals with quasilinear preferences for two goods: one excludable private good and one non-excludable and non-rivaling public good Z. Since the public good must be paid for by the private good, individual preferences can be written as:

$$
\begin{equation*}
W_{i}=\theta^{\prime} v(Z)-\alpha Z \tag{2.1}
\end{equation*}
$$

where the parameter $\theta^{i}>0, \alpha$ is i's share of the total production-cost, and $v$ is an increasing and concave function satisfying the following conditions:
(i) for all $Z$ : $v^{\prime}(Z)>0$ and $v^{\prime}{ }^{\prime}(Z)<0$. (ii) $\lim _{Z \rightarrow 0}{ }^{\prime}(Z)=\infty$. (iii) $\lim _{Z \rightarrow \infty} v^{\prime}(Z)=0$. (iv) $\mathrm{v}(0)=0$.

In any coalition, majority voting determines the production level, and the cost is equally distributed among the citizens. Thus, the median voter m in a coalition I plays a non-cooperative contribution game with other regions -I. The median voter j in region/coalition j wants to solve:
$\operatorname{Max}_{Z_{j}} \theta_{j} v\left(Z_{j}+Z_{-j}\right)-\frac{1}{n_{j}} Z_{j}$

3 Compared to Ellingsen (1998), this model is limited to the case of homogenous populations in each province, having positive preferences for the public good.
giving the first-order condition:
$n_{j} \theta_{j} v^{\prime}(Z) \leq 1,=$ if $Z_{j}>0$
where $n_{j}$ is population in region/coalition $j$. I will assume that all $n_{j} \theta_{j}$ differ for all j . From (2.3) it then follows that only one j is producing in equilibrium. ${ }^{4}$ The other provinces are free-riding and satisfied with this production level. If I is a producing region, (2.3) becomes:
$n_{I} \theta_{m} v^{\prime}\left(Z^{I}\right)=1 \Rightarrow Z^{I}=v^{\prime-1}\left(\frac{1}{n_{I} \theta_{m}}\right)$
Where $\mathrm{v}^{\prime-1}$ is a decreasing function, i.e. $\mathrm{Z}^{\mathrm{I}}$ is increasing in $\mathrm{n}_{\mathrm{I}} \theta_{\mathrm{m}}$.
There is clearly an externality, since other regions also benefit from $Z^{1}$. The welfare is:
$W_{m}^{I}=\theta_{m} v\left(Z^{I}\right)-\frac{1}{n_{I}} Z^{I}$
$W_{j}^{I}=\theta_{j} v\left(Z^{I}\right) \quad$ for $\quad j \notin I$
Let $A$ be the region with the largest $n_{j} \theta_{j}$, i.e. only $A$ is producing in the separating equilibrium ( S ), and

$$
\begin{equation*}
Z^{S}=v^{\prime-1}\left(\frac{1}{n_{A} \theta_{A}}\right) \tag{2.7}
\end{equation*}
$$

Comparing with (2.4) we see that:

$$
\begin{equation*}
\operatorname{sign}\left(Z^{I}-Z^{S}\right)=\operatorname{sign}\left(n \theta_{m}-n_{k} \theta_{k}\right) \tag{2.8}
\end{equation*}
$$

So production under integration may be larger or smaller than under separation.
A's increase in utility when $A$ is forming the coalition I instead of having $S$ is:

$$
\begin{equation*}
\Delta W_{A}^{I} \equiv W_{A}^{I}-W_{A}^{S}=\theta_{A} \Delta v^{I}+\frac{1}{n_{A}} Z^{S}-\frac{1}{n_{I}} Z^{I} \tag{2.9}
\end{equation*}
$$

where $\Delta v^{t}=v\left(Z^{t}\right)-v\left(Z^{v}\right)$. For A, the benefit of integration is cost sharing. However, there can also be a cost of integration for $A$, if the new median voter is very different from $\theta_{\mathrm{A}}$. The larger the coalition is (the more cost sharing), the more different $\theta_{\mathrm{m}}$ can be, for integration still to be beneficial for A .

If another region j integrates with A , its change in welfare is:
$\Delta W_{j}^{I}=\theta_{j} \Delta v^{I}-\frac{1}{n_{I}} Z^{I}$
which is increasing in $\theta_{j}$ and $n_{I} / n_{A}$. The cost of integrating is the cost-share. The potential benefit is that together the coalition can afford a larger Z .

[^0]The positive analysis of separation vs. integration looks for the sub game perfect equilibrium (SPE) in a two-stage game. In the first stage, each region decides whether it should integrate or not by majority voting. Integration can only occur if the majority in all integrating regions wants to integrate. In the second stage, the level of production within any jurisdiction is determined by majority voting. Since the preferences in (2.1) are single-peaked, the median voter in A or in I decides the production level. Integration occurs if it is a SPE of the game. The individuals are assumed to be rational, perfectly foresighted and utility maximizing. Integration then requires that $\Delta W_{j} \geq \mathrm{U}$ for the median voter in all regions j . Social optimality is therefore a necessary, but not a sufficient, requirement for integration to be equilibrium.

## Two regions

Suppose $n_{A} \theta_{A}>n_{B} \theta_{B}$, s.t. only A produces in separation because of it's stronger preferences for the public good. If $\mathrm{n}_{\mathrm{A}}>\mathrm{n}_{\mathrm{B}}$, A will (because it has majority) determine the level of public good also in integration, and integration will only mean cost-sharing. A will then always prefer integration. B will prefer integration only if Z will increase sufficiently. From (2.10) it is more likely that $B$ wants to integrate the larger is $\theta_{\mathrm{B}} / \theta_{\mathrm{A}}$ and $\mathrm{n}_{\mathrm{B}} / n_{\mathrm{A}}$. Ellingsen shows that if $\theta_{\mathrm{B}}=\theta_{\mathrm{A}}$, integration is optimal but will always be resisted by a sufficiently small region, which prefer to free ride. If $\mathrm{n}_{\mathrm{A}}<\mathrm{n}_{\mathrm{B}}$, the median voter in integration is in B. Thus, A prefers to integrate unless B's preferences for Z are sufficiently different. But $n_{A} \theta_{A}>n_{B} \theta_{B}$ implies that $\theta_{A}>\theta_{B}$, making it less likely that B wants to integrate.

To be more specific about when the different cases appear, it is necessary to specify the function v . This is valuable as an example, both to check and illustrate propositions as well as to gain intuitive understandings. In the special case $v(Z)=\sqrt{ } Z$, figure 1 shows SPE and social optimal regimes. It turns out that only the relative differences in size $x=n_{A} / n_{B}$ and preferences $\mathrm{y}=\theta_{\mathrm{B}} / \theta_{\mathrm{A}}$ are important.


Figure 1

A couple ( $\mathrm{i}, \mathrm{j}$ ) denotes that i is the optimal regime, while j is the SPE. A similar figure appears in Ellingsen (1998) and Appendix shows how this is found. Note that $n_{A} \theta_{A}>n_{B} \theta_{B}$ implies that $x>y$.

## 3. Three regions

There are two natural generalizations of the two-country model. One is to allow for more than two regions. The other is to allow the public good to be less perfect: It might not be realistic that the benefit of the public good is independent of the location of the production. A generalization could be to introduce local public goods, and parameters $\mathrm{c}_{\mathrm{ij}}$ denoting the marginal contribution at location i of production at location j . The intuition of this is captured by looking at two extremes. In this section we consider the first extreme where the public good is perfectly available for all. Next section discusses the other extreme where only the closest neighbor can benefit from the production.

From (2.3) we know that under separation, only one region will produce Z. Let this, again, be region A. Then:

$$
\begin{align*}
& n_{A} \theta_{A}>n_{B} \theta_{B}, n_{C} \theta_{C}  \tag{3.1}\\
& \mathrm{Z}^{S} \text { is again given by: }
\end{align*}
$$

$$
\begin{equation*}
n_{A} \theta_{A} v^{\prime}\left(Z^{S}\right)=1 \Rightarrow Z^{S}=v^{\prime-1}\left(\frac{1}{n_{A} \theta_{A}}\right) \tag{2.7}
\end{equation*}
$$

As will be discussed, several new possible situations now arise: B may resist to integrate in the hope that C instead will integrate with A ; B may choose to integrate because it expects C also to do so; B may choose to integrate if and only if C also does so, and A may want to integrate with only one of B and C. If we first discuss B and C's decisions, the following normal form game is helpful:

|  | $\mathrm{C}: \mathrm{S}$ | $\mathrm{C}: \mathrm{I}$ |
| :--- | :--- | :--- |
| B: S | $\mathrm{W}^{\mathrm{S}}$ | $\mathrm{W}^{\mathrm{AC}}$ |
| B: I | $\mathrm{W}^{\mathrm{AB}}$ | $\mathrm{W}^{\mathrm{F}}$ |

Game 1: The integration game between region $B$ and $C$
$W^{R}$ is the vector $\left(W_{B}{ }^{R}, W_{C}{ }^{R}\right)$ where $R$ denotes regime. $A B$ means (partly) integration between A and B and F means (full) integration between $\mathrm{A}, \mathrm{B}$ and C .

B's welfare in these cases is:

$$
\begin{align*}
& W_{B}^{S}=\theta_{B} v\left(Z^{S}\right)  \tag{3.2}\\
& W_{B}^{A C}=\theta_{B} v\left(Z^{A C}\right) \tag{3.3}
\end{align*}
$$

$$
\begin{align*}
& W_{B}^{A B}=\theta_{B} v\left(Z^{A B}\right)-\frac{1}{n_{A}+n_{B}} Z^{A B}  \tag{3.4}\\
& W_{B}^{F}=\theta_{B} v\left(Z^{F}\right)-\frac{1}{n_{A}+n_{B}+n_{C}} Z^{F} \tag{3.5}
\end{align*}
$$

C will only integrate with A if it makes Z increase, therefore we must have $W_{B}^{J}<W_{B}^{A C}$ : B always benefits when A and C integrate. I will also assume that $W_{B}^{A D}<W_{B}^{r}$ : B always benefits if C is included in the integration, and that A must participate in any coalition. This is done to limit the number of cases, and it leaves us with six possible preference orderings for B :

Type 1: $W_{B}^{A B}<W_{B}^{F}<W_{B}^{S}<W_{B}^{A C}$
In this case B prefers not to integrate whatever C does, and separation is preferred even to full integration. From the previous section we know that to have $W_{B}^{A B}<W_{B}^{S}, \theta_{\mathrm{B}}$ must be small and $\mathrm{n}_{\mathrm{A}} / \mathrm{n}_{\mathrm{B}}$ large. To also have $W_{B}^{F}<W_{B}^{S}$, also $\mathrm{n}_{\mathrm{A}} /\left(\mathrm{n}_{\mathrm{B}}+\mathrm{n}_{\mathrm{C}}\right)$ must be large: not even full integration will increase Z a lot. Clearly, strategy S in game 1 is B 's dominant strategy. If also $\theta_{\mathrm{C}}$ is moderate, C may be of the same type and the SPE of the game (the Nash Equilibrium in game 1 ) is $(S, S)$ giving the regime $S$.

Type 2: $W_{B}^{A B}<W_{B}^{S}<W_{B}^{F}<W_{B}^{A C}$
Also in this case, $\theta_{\mathrm{B}}$ and $\mathrm{n}_{\mathrm{A}} / \mathrm{n}_{\mathrm{B}}$ are so small that B prefers S to AB . However, $\mathrm{n}_{\mathrm{B}}+\mathrm{n}_{\mathrm{C}}$ is large enough to make it worthwhile to integrate with A if C does the same: then Z will increase sufficiently. Compared to type 1 , this requires a larger $n_{B}+n_{C}$ or $\theta_{B}$. However, $S$ in figure 2 is still the dominant strategy. If the case is similar for C , it is a prisoner dilemma (PD) game: both prefer the other region to integrate (for a larger Z ), but none wants to integrate. The unique SPE is separation although both would benefit from full integration.

Proposition 1: If $n_{B}$ and $n_{C}$ both are small, but $\theta_{B}$ and $n_{B}+n_{C}$ are sufficiently large, $B$ is of type 2. If also $C$ is of type 2, game 1 is a PD-game. To reach the beneficial regime $F$, bilateral commitment is needed.

Type 3: $W_{B}^{A B}<W_{B}^{S}<W_{B}^{A C}<W_{B}^{F}$
In this case, B wants to integrate if and only if C also does so. The individual cost of production is decreasing less if $B$ joins $A C$, than if $B$ joins $A$ only (since $\left.n_{B} / n_{A}>n_{B} /\left(n_{A}+n_{C}\right)\right)$. To be of this type, therefore, it is required that $B$ is not determining the level of the production in the coalition AB , while the level of production is more like B 's preferences under the regime F . It is therefore required that $n_{B}+n_{C}>n_{A}>n_{B}, n_{C}$. If $C$ is of same type, the game is a coordination (CO) game, with two SPE: $S$ and $F$. Both B and C prefer $F$ to $S$. If C is of type 2, C will not integrate and the unique equilibrium is S . However, $C$ would like to make it credible for $B$ that $C$ wants to integrate. One way to do this is that $A$ and $C$ integrate first, and $B$ follows.

Proposition 2: If $n_{B}+n_{C}>n_{A}>n_{B}, n_{C}$, and $\theta_{B}$ is large, $B$ is likely to be of type 3 . If also $C$ is of type 3, game 1 is a $C O$-game with $F$ as the best outcome. If $B$ is of type 3 and $C$ of type $2, C$ will prefer to cheat (not integrate), and $S$ is the outcome unless $C$ can commit to integrate.

Type 4: $W_{B}^{\nu}<W_{B}^{A D}<W_{B}^{\Gamma}<W_{B}^{A C}$
In this case, B wants to integrate with A , if and only if C does not integrate with A alone. Moreover, B prefers A and C to integrate instead of integrating with A itself. To have $W_{B}^{\nu}<W_{B}^{A D}, \mathrm{n}_{\mathrm{B}} / \mathrm{n}_{\mathrm{A}}$ and $\theta_{\mathrm{B}}$ must not be too small. To have $W_{B}^{r}<W_{B}^{A c}$, adding B to AC should not increase Z very much, i.e. $\mathrm{n}_{\mathrm{B}} /\left(\mathrm{n}_{\mathrm{A}}+\mathrm{n}_{\mathrm{C}}\right)$ must be small. If both B and C are of type 4 , game 1 is a hawkdove (HD) game: there are two SPE and conflicting interests between $B$ and C.

Proposition 3: If $n_{B} / n_{A}$ is large but $n_{B} /\left(n_{A}+n_{C}\right)$ small, $B$ is likely to be of type 4. If also $C$ is of type 4, game 1 is a HD-game.

Type 5: $W_{B}^{S}<W_{B}^{A B}<W_{B}^{A C}<W_{B}^{F}$
In this case, $B$ wants to integrate whatever $C$ does. The difference to type 4 is that $B$ integrates even if $C$ does. Compared to type 4 , this requires a larger $n_{B}$; a smaller $n_{C}$ (if for example $n_{A}+n_{C}$ is not much larger than $n_{A}$ ); a larger $\theta_{B}$; or that B and C are of similar types and together larger than A . If C is of same type, the unique SPE in game 1 is F .

Type 6: $W_{B}^{S}<W_{B}^{A C}<W_{B}^{A B}<W_{B}^{F}$
The difference to type 5 is that B prefers integration with A alone, to the situation when only $A$ and $C$ integrate. This may be the case if $\theta_{B}$ and $n_{B} / n_{A}$ are large, while $\mathrm{n}_{\mathrm{C}} / \mathrm{n}_{\mathrm{A}}$ is small. This obviously require that $\mathrm{Z}^{\mathrm{AB}}>\mathrm{Z}^{\mathrm{AC}}$, thus: both $B$ and $C$ cannot be of type 6 . As for type 5 , the dominant strategy is $I$.

This leaves us with 35 cases for B and C's types. The corresponding equilibria in game 1 are shown in the table below. Regimes in parenthesis show the outcome if B and C can make commitments.

A's type B's type

|  | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | S | S | S | AC | AC | AC |
| 2 | S | $\mathrm{~S},(\mathrm{~F})$ | $\mathrm{S},(\mathrm{F})$ | AC | AC | AC |
| 3 | S | $\mathrm{~S},(\mathrm{~F})$ | $\mathrm{S}, \mathrm{F}$ | F | F | F |
| 4 | AB | AB | F | $\mathrm{AB}, \mathrm{AC}$ | AC | AC |
| 5 | AB | AB | F | AB | F | F |
| 6 | AB | AB | F | AB | F |  |

## Table 1

In the special case where $v(Z)=\sqrt{Z}, \theta_{B}=\theta_{C}=y \theta_{A}, \quad n_{B}=n_{C}=n_{A} / x$, $B$ and C's common type can be (see Appendix) illustrated as a function of $x$ and $y$ :


Figure 2
The numbers shows B and C's common type as a function of $x$ and $y$. Type 3 and 6 are no possible in this special case.

A's preferences have been neglected so far in this analysis. It shouldn't be, since interesting cases would be lost. As long as A is in majority it is deciding the production level, and A will always be happy to share the production costs with somebody. If - trough an integration - A is losing its majority, A will no longer decide the production level, and might not benefit from the integration. Obviously, A is benefiting from losing the majority only if the benefit of sharing the costs of production is larger than the cost of letting other decide the production level. This latter cost is larger, the more different A's type is to B and C's types.

Proposition 4: If, $n_{B}>n_{A}, A$ will prefer $S$ to $A B$ if their types are very different. If $n_{A}>n_{B}+n_{C}$, A prefer as large a coalition as possible. If $n_{B}+n_{C}>n_{A}>n_{B}$, and if their types are very different, A prefer $A B$ to $F$.

In the same special case as in figure 2, A's preferences are as follows:


Figure 3

In area $1, W_{A}{ }^{A B}>W_{A}{ }^{S}>W_{A}{ }^{F}, A$ only wants to integrate with one other region, and would prefer separation to full integration. The reason is that B and C are too different, and together they are in majority. In area 2, $\mathrm{W}_{\mathrm{A}}{ }^{\mathrm{AB}}>\mathrm{W}_{\mathrm{A}}{ }^{\mathrm{F}}>\mathrm{W}_{\mathrm{A}}{ }^{\mathrm{S}}$, A prefers partly integration, but accepts full integration if separation is the alternative. In area $3, \mathrm{~W}_{\mathrm{A}}{ }^{\mathrm{F}}>\mathrm{W}_{\mathrm{A}}{ }^{\mathrm{AB}}>\mathrm{W}_{\mathrm{A}}{ }^{\mathrm{S}}$, A wants as large coalition as possible. For $x<2$, this is because $B$ and $C$ have sufficiently similar preferences. For $x>2, A$ is determining the production level anyway. In the black area, $\mathrm{W}_{\mathrm{A}}{ }^{\mathrm{S}}>\mathrm{W}_{\mathrm{A}}{ }^{\mathrm{F}}>\mathrm{W}_{\mathrm{A}}{ }^{\mathrm{AB}}$ : A prefers separation. The reason is that A is smallest and the majority will determine the production-level in any integration. Such a low y implies that the majority don't care much about Z, and will choose too little production in A's view. In the gray area, $\mathrm{W}_{\mathrm{A}}{ }^{\mathrm{F}}>\mathrm{W}_{\mathrm{A}}{ }^{\mathrm{S}}>\mathrm{W}_{\mathrm{A}}{ }^{\mathrm{AB}}$, A prefers full integration, while separation is better than partly integration. In any integration, the majority has little preferences for Z , but under full integration more is being produced. Appendix shows how the curves in figure 3 are found.

## 4. Three regions in a line

Now assume that the three regions A, B and C are lying next to each other:


Figure 4
Assume that only the closest neighbor is benefiting from the production in any region. It is then important where Z is produced, not only who that is paying for the production. In separation, the first-order conditions (2.3) of maximization become:
$\mathrm{n}_{\mathrm{B}} \theta_{\mathrm{B}} \mathrm{v}^{\prime}\left(\mathrm{Z}_{\mathrm{A}}^{\mathrm{S}}+\mathrm{Z}_{\mathrm{B}}^{\mathrm{S}}+\mathrm{Z}_{\mathrm{C}}^{\mathrm{S}}\right) \leq 1,=$ if $\mathrm{Z}_{\mathrm{B}}^{\mathrm{S}}>0$
$n_{j} \theta_{j} v^{\prime}\left(Z_{j}^{S}+Z_{B}^{S}\right) \leq 1,=$ if $Z_{j}^{S}>0, j=A, C$
One possible solution of (4.1) and (4.2) is that only $B$ is producing. However, this requires that:
$n_{B} \theta_{B} \geq \max \left\{n_{A} \theta_{A}, n_{C} \theta_{C}\right\} \equiv k$

Another possibility is that only A and C produce. For this to be an equilibrium, it requires that $B$ is satisfied with $Z_{A}+Z_{C}$, because otherwise, it would have taken the cost of producing more itself. That is, (4.1) must hold, implying:

$$
\begin{equation*}
n_{B} \theta_{B} \leq \frac{1}{v^{\prime}\left(v^{\prime-1}\left(\frac{1}{n_{A} \theta_{A}}\right)+v^{\prime-1}\left(\frac{1}{n_{C} \theta_{C}}\right)\right)} \equiv l \in\left(\max \left(n_{A} \theta_{A}, n_{C} \theta_{C}\right), n_{A} \theta_{A}+n_{C} \theta_{C}\right) \tag{4.4}
\end{equation*}
$$

Which means that $n_{B} \theta_{B}$ can not be too large for this to be an equilibrium. In the case where $v(Z)=\sqrt{Z}$ we get:
$l=\sqrt{\left(n_{A} \theta_{A}\right)^{2}+\left(n_{C} \theta_{C}\right)^{2}}$

Proposition 5: If $n_{B} \theta_{B}<k$, the only equilibrium under separation is that only $A$ and $C$ are producing. If $n_{B} \theta_{B}>l$, the only equilibrium is that only $B$ is producing. If $k<n_{B} \theta_{B}<l$, both these equilibria are possible.

For comments, see Appendix. The SPE of the game will depend on the initial Nash-equilibrium. I therefore divide the discussion under two main headings: one for each set of Nash-equilibria.

Integration can occur in three ways: B can integrate with one of its neighbors; all can integrate; or only A and C may integrate. It is important where in the jurisdiction production occur. If some regions integrate, they can determine where the production should be if the (cost-) shares are tradable. Then all production can be in one region, while the other regions pay their share in cash. For many public goods, it is likely that the shares are not tradable: the only reasonable way a region can pay their share of the cost, is to produce their share of the amount. For e.g. environmental agreements on pollution quotas, the shares are tradable if and only if the quotas are tradable.
$Z_{B}^{S}=0:$

When $\mathrm{Z}_{\mathrm{B}}$ is zero, then from (4.2), both $\mathrm{Z}_{\mathrm{A}}$ and $\mathrm{Z}_{\mathrm{C}}$ must be positive. This is a good situation for B : B enjoys production from both its neighbors, and since v is concave, B is less willing to integrate with A compared to the previous section. A's welfare improvement of integration with $B$ is exactly as discussed in section 3, and A is indifferent whether the production is in A or in B . $B$ is not: If all production in an integration with $A$ is in $A, C$ will still produce as much as it would in separation. If there is also some production in $B$, C will reduce its production according to (4.2). Therefore, B is less likely to benefit from an integration with A if the production shares are not tradable.

If the shares are tradable, a full integration generates most welfare if all production is in B. A's incentives for full integration is then exactly as in section 3. B, however, enjoys both A's and C's production in separation and is less likely to benefit from full integration compared to section 3. Moreover, B might prefer partly integration with A only, because then C must produce as well. In this case $B$ wants to exclude $C$ from the coalition. If the shares are not tradable, A and C cannot enjoy each other's production and their benefit from full integration is less than in section 3 .

For A and C to benefit form the coalition AC, they must lay the production in B . Total production is then likely to decrease, compared to separation, and B will then loose on the integration. B will then have incentives to prevent A and C from integrating. All this can be summarized in a proposition.

Proposition 6: $B$ is less likely to form a coalition with $A$ or with AC, if B has another producing neighbor $C$. If then production shares are not tradable, $B$ is even less willing to integrate with $A$, and $A$ and C's benefit from full integration is smaller. If production shares are tradable, $B$ might want to exclude $C$ from integrating with $A$ or $A B$.

$$
Z_{B}^{S}>0
$$

Again only one region (now labeled $B$ ) is producing in equilibrium while $A$ and C free ride. Suppose A and B integrate. If production shares are tradable, $B$ will prefer to lay the production in $A$, such that also $C$ must start production according to (4.2). B's incentives to exclude $C$ from the coalition $A B$ are then as above. C will in this case lose on the integration between A and $B$. It may then happen that $A$ and $C$ will compete to integrate with $B$, even if f.ex. C would not have integrated with B if A did not exist. Alternatively, A and $C$ might collude to reduce $B$ 's power.

If production shares are not tradable, both B and C is likely to benefit from a coalition $A B$. C's incentives to integrate with $A B$ is then as above.

If we again change the labels on $A$ and $B$, we can summarize this as:
Proposition 7: Suppose only A has a neighbor $C$ and the shares are tradable. $A$ is then likely to exclude $B$ from integrating with $A C$, and $B$ will loose on the coalition $A C$. Therefore, the existence of $C$ makes $B$ more willing to integrate with $A$.

## 5. Final Remarks

The simple model in this paper manages to explain a wide range of observable and interesting real-world situations. In making predictions and explanations it is important to know when the different cases appear, and the paper characterizes these in terms of relative differences in size and preferences across the regions. One drawback in the paper is that specific characterizations of the different cases are only available for specific utility functions for the public good. To more specifically characterize the different cases in the general case, more research should be done. In reality, integration and separation is determined by a huge number of forces, in addition to the one described here. A whole research program is open to study the interactions of these forces with the effects of differences in size and preferences.

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## Appendix

Let $v(Z)=\sqrt{Z}$. From (2.3):
$v^{\prime}(~)=\frac{1}{2 \sqrt{Z}} \quad Z=\left(\frac{1}{2{ }^{\prime}(Z)}\right)^{2}=\left(\frac{n_{I m}}{2}\right) \quad(Z)=\frac{n \theta_{m}}{2}$ (A1)
$\mathrm{n}_{\mathrm{B}} \mathrm{n}_{\mathrm{C}} \quad \theta_{\mathrm{A}}$
$x=\frac{n_{A}}{n_{B}} \Rightarrow \frac{n_{A}}{n}=\frac{x}{1+x}, \frac{n_{B}}{n}=\frac{1}{1+x}$ (A2)
$y=\frac{\theta_{B}}{\theta_{A}} \Rightarrow \frac{\theta_{A}}{\bar{\theta}}=\frac{1+x}{y+x}, \frac{\theta_{B}}{\bar{\theta}}=\frac{y(1+x)}{y+x}$ (A3)
$\theta_{A} n_{A}>\theta_{B} n_{B} \Rightarrow x>y$
In separation (S), (2.4), (2.5) and (2.6) give:
$Z_{A}^{S}=Z^{S}=\left(n_{A} / 2\right)^{2}$ (A4)
$W_{A}^{S}=\frac{n_{A}}{4}$ (A5)
$W_{j}^{S}=\frac{n_{A} y}{2}, j=B, C(\mathrm{~A} 6)$

If $A$ and $B$ are integrating, and $x>1, A$ is in majority and contains $m$. Then:
$Z^{A j}=\left(\frac{n_{A}+n_{j}}{2}\right)^{2}, j=B, C(\mathrm{~A} 7)$
$W_{A}^{A B}=\frac{\left(n_{A}+n_{B}\right)}{4}$ (A8) $W_{B}^{A B}=\frac{\left(n_{A}+n_{B}\right)}{4}(2 y-1)$ (A9)
$W_{C}^{A B}=\frac{\left(n_{A}+n_{B}\right) y}{2}(\mathrm{~A} 10)$

If $\mathrm{x}<1, \mathrm{~m}$ is in region B , and determines the production level:

$$
Z^{A B}=\left(\frac{n_{A}+n_{B}}{2} y\right)^{2}
$$

$$
\begin{equation*}
W_{A}^{A B}=\frac{\left(n_{A}+n_{B}\right)(2-y) y}{4} \quad W_{B}^{A B}=\frac{\left(n_{A}+n_{B}\right) y^{2}}{4} \tag{A13}
\end{equation*}
$$

$W_{C}^{A B}=\frac{\left(n_{A}+n_{B}\right) y^{2}}{2}$

Symmetrically under regime AC.
If there is full integration (F), and $\mathrm{x}<2$, a median voter in B or C is deciding the production level. Then:
$Z^{F}=\left(\frac{n y}{2}\right)^{2}$
$W_{A}^{F}=\frac{n y(2-y)}{4}$ (A16) $W_{B}^{F}=W_{C}^{F}=\frac{n y^{2}}{4}$ (A17)
If $x>2$, $A$ is still having majority. Then:
$Z^{F}=\frac{n^{2}}{4}$ (A18)
$W_{A}^{F}=\frac{n}{4}(\mathrm{~A} 19) W_{B}^{F}=W_{C}^{F}=\frac{n}{4}(2 y-1)(\mathrm{A} 20)$
Comparing all these, we find that for $\mathrm{x}<1$ :
$W_{B}^{A B}-W_{B}^{S} \geq 0 \Leftrightarrow y \geq \frac{2 x}{1+x}$ (A21)
$W_{B}^{F}-W_{B}^{A C}<0$ always (A22)
$W_{B}^{F}-W_{B}^{S} \geq 0 \Leftrightarrow y \geq \frac{x}{2+x}$ (A23)
For $1<x<2$ :
$W_{B}^{A B}-W_{B}^{S} \geq 0 \Leftrightarrow y \geq \frac{1}{2}+\frac{x}{2}$ (A24)
$W_{B}^{F}-W_{B}^{A C} \geq 0 \Leftrightarrow y \geq \frac{2+2 x}{2+x}$ (A25)
$W_{B}^{F}-W_{B}^{S} \geq 0 \Leftrightarrow y \geq \frac{x}{2+x}$ (A26)
And for $\mathrm{x}>2$ :
$W_{B}^{A B}-W_{B}^{S} \geq 0 \Leftrightarrow y \geq \frac{1}{2}+\frac{x}{2}$ (A27)
$W_{B}^{F}-W_{B}^{A C} \geq 0 \Leftrightarrow y \geq 1+\frac{x}{2}$ (A28)
$W_{B}^{F}-W_{B}^{S} \geq 0 \Leftrightarrow y \geq \frac{1}{2}+\frac{x}{4}$ (A29)
A21-A29 are the curves in figure 3.
Comparing A's utility in the different cases, we find:
If $\mathrm{x}<1$ :
$W_{A}^{A B}-W_{A}^{S} \geq 0 \Leftrightarrow y \in\left(1-\sqrt{\frac{1}{1+x}}, 1+\sqrt{\frac{1}{1+x}}\right)$
$W_{A}^{F}-W_{A}^{A B} \geq 0 \Leftrightarrow y(2-y) \geq 0(\mathrm{~A} 31)$
$W_{A}^{F}-W_{A}^{S} \geq 0 \Leftrightarrow y \in\left(1-\sqrt{\frac{2}{2+x}}, 1+\sqrt{\frac{2}{2+x}}\right)$
If $1<x<2$
$W_{A}^{A B}-W_{A}^{S}>0$ always (A33)
$W_{A}^{F}-W_{A}^{A B} \geq 0 \Leftrightarrow y \in\left(1-\sqrt{\frac{1}{2+x}}, 1+\sqrt{\frac{1}{2+x}}\right)$
$W_{A}^{F}-W_{A}^{S} \geq 0 \Leftrightarrow y \in\left(1-\sqrt{\frac{2}{2+x}}, 1+\sqrt{\frac{2}{2+x}}\right)$
If $\mathrm{x}>2$ :
$W_{A}^{A B}-W_{A}^{S}>0$ always (A36)
$W_{A}^{F}-W_{A}^{A B}>0$ always $(\mathrm{A} 37)$
$W_{A}^{F}-W_{A}^{S}>0$ always (A38)
A30-A38 are the curves drawn in figure 4.

## Comments to Proposition 5:

Can there also be other equilibria fulfilling (4.1) and (4.2)?
Suppose $Z_{A}^{S}=0$. Then also $Z_{C}^{S}=0$ when $n_{B} \theta_{B} \neq n_{C} \theta_{C}$. It is therefore not possible that B and only one of the two other regions produce in equilibrium. But (4.1) and (4.2) are three equations that always have a solution for the three unknown. These solutions are positive only if both (4.3) and (4.4) hold. Even then, it is very unlikely that there is production in all regions, it is easy to see that such a solution is unstable: suppose all regions are producing such that all (4.1) and (4.2) hold with equality. If A increases its production a marginal amount, B will reduce its production by the same amount (for (4.1) to hold). Then, C will increase its production by the same amount, B will reduce again, and A will also increase, and so on. The end of this story is the equilibrium where only A and C produce. If it were B that increased its production a marginal amount, the end of the story would be that only B produces. There are therefore two likely Nash-equilibria. It is probably most likely that (4.3) does not hold (because there are two regions B can compare its preferences with).

Then, the unique solution is that only A and C produces. If (4.4) does not hold, the unique solution is that only B produces. When both (4.3) and (4.4) hold, both these two solutions are Nash-equilibria.


[^0]:    4 This is due to the assumptions of perfect spillovers in (2.2) and the quasilinear utility function. More modest assumptions will still lead to qualitatively similar results.

