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Modifying the Stolper-Samuelson Relationship

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Modifying the Stolper-Samuelson Relationship

Arne Melchior

[Abstract] From the Stolper-Samuelson Theorem, it is expected that North-South trade reduces the real wage of unskilled labour in the North. This paper questions the underlying assumption that trading countries are diversified, and examines theoretically the trade-wage link when the South is completely specialised. While it remains true that trade with the South negatively affects wages in the North, it is no longer the case that the poorer the trade partner is, the more harmful is trade for Northern wages. The negative wage impact is largest when the South has an intermediate capital-labour ratio, since it is then a more efficient producer. This also gives the largest aggregate welfare gains from trade in the North. The specialised South also gains from trade, and these gains are relatively larger, the more extreme is its factor composition. But even if the poorest countries gain from trade, capital accumulation may be more important for their welfare.

Keywords International trade, neoclassical trade theory, trade and wages, North-South trade, the HOS model.

JEL classification code: F11, F16.

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1. Introduction[1]

In the OECD, an increased wage gap between skilled and unskilled labour has been observed in some countries, and more unemployment among the unskilled in other ones. A major issue in recent international trade research has been whether this "skill gap" is caused by rising North-South trade, or by technological change that increases the demand for skills (for surveys, see e.g. Burtless 1995, Feenstra et al. 2004).

In empirical work on the trade-wage link, the theoretical foundation has mainly been the Theorem of Stolper and Samuelson (1941) (see e.g. Deardorff and Hakura 1994). Stolper and Samuelson concluded: "International trade necessarily lowers the real wage of the scarce factor expressed in terms of any good" (ibid., 346). According to this, trade with the labour-abundant South will lead to a reduction in the real wage of labour in the North. There is a corresponding "magnification effect" by which the relative change in factor prices is stronger than the relative change in goods prices (Jones 1965). In textbooks, it is the latter relationship that is presented as the Stolper-Samuelson Theorem (SST).

It is common knowledge in the literature on the HOS (Heckscher-Ohlin-Samuelson) trade model that SST only applies when countries are diversified; e.g. produce both goods in the 2x2x2 model. Stolper and Samuelson (1941, 352) explicitly considered the case with complete specialisation and concluded that for a capital-abundant country, trade would reduce the real wage measured in terms of the exported good, but the impact expressed in terms of the imported good would be ambiguous. In general, the 60-year old later literature on the HOS model has not followed up sufficiently this interest for cases with complete specialisation. The focus has rather been on factor price equalisation (FPE), which also rests on the assumption of diversified production. As noted by Bhagwati et al. (1998, 107), "incomplete specialisation in equilibrium is considered the norm and complete specialisation … is treated as an unlikely event".

The theoretical literature has focused extensively on the *conditions* for FPE to occur, and a sizeable literature has asked whether FPE is possible with various modifications and extensions of the 2x2x2 HOS model (see e.g. Deardorff (1994), or Jones and Neary (1984) for a survey). Recently, Thompson (2004) has examined the robustness of SST to various modifications of the HOS model. In spite of this large literature providing ample knowledge about when FPE occurs and when SST applies, the literature on what happens if there is not FPE, and SST does not apply, is still limited.

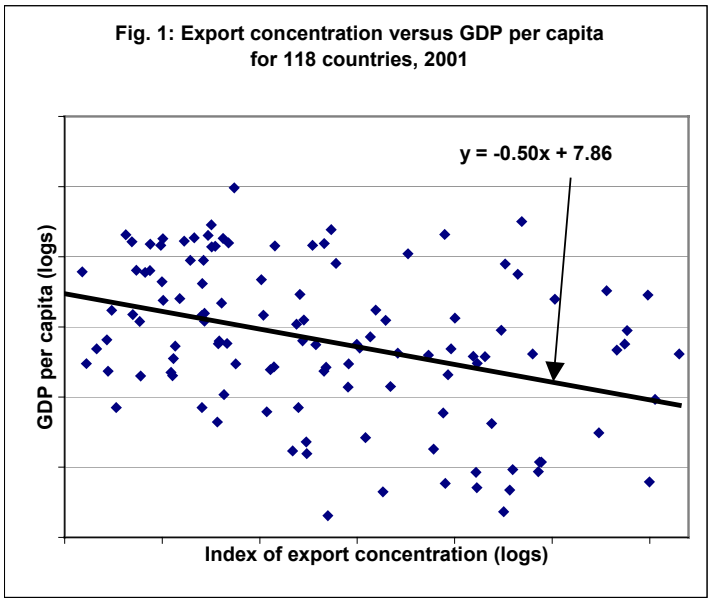
In recent years, however, research has started to fill this gap by examining trade and specialisation in cases with complete specialisation, and there has been more attention to the issue (see e.g. Leamer (1984, 19) or Wood (1994, 29)). Chipman (1992) shows how the HOS model with complete specialisation can contribute to the explanation of intra-industry trade. Some authors have used multi-good, multi-country HOS models to examine trade and growth when countries at different income levels form clusters or specialise in different

[1] I thank colleagues at NUPI, in particular Hege Medin and Leo Andreas Grünfeld, for useful comments to an earlier draft.

"diversification cones", with FPE within each cone, producing a distinct subset of goods (Davis 1996, Deardorff 2000, 2001a, 2001b, Schott 2003).

Hence there is now more knowledge about international trade and specialisation in cases with complete specialisation. On the behaviour of goods and factor prices in such cases, however, research is still limited. Contributions do exist; Deardorff (2001b) e.g. also examines factor prices in a situation where two countries are specialised in different goods, and showed e.g. that the factor price ratio is then strongly influenced by the demand shares for the different goods. If all countries are completely specialised, we show here that their income levels and utility will depend mainly on the demand share for their particular product. Since there is no room for factor substitution across goods, terms-of-trade effects dominate the scene. In the case when some countries remain diversified, the scope for factor substitution in some countries remains and makes this an intermediate case between the standard HOS model and the case with complete specialisation in all countries. The analysis also suggests that complete specialisation in both countries is less likely than specialisation in one of them only. We therefore extend the analysis to the case is when some countries are completely specialised and others not. Although we also examine the outcome when both countries are completely specialised, our main focus will be on the "mixed case".

This theoretical focus also has an empirical motivation: Poor countries are on average less diversified than rich ones. As an illustration, Figure 1 plots an index of sectorial export concentration against GDP per capita.[2]



The exports of rich countries are on average more diversified. A similar pattern obtains for large countries (measured by GDP). Including both variables in a

[2] Trade data are from the COMTRADE database, and income data are from World Bank (2004). The concentration index has the form $H_i=\Sigma_k s_{ik}^2$, $0<H_i<1$, where s_{ik} is the share of sector k in country i's total exports; with sectors defined as the 96 two-digit chapters in the HS1996 trade classification. PPP income data in current USD are applied. We use trade data due to the more limited availability of disaggregated production data for many countries.

simple regression, we obtain (with P values as subscripts, adjusted R^2=0.20, N=118): $\ln(H_i) = 2.57_{0.0021} - 0.12_{0.0016} \ln(GDP_i) - 0.18_{0.0088} \ln(\text{GDP per capita}_i)$. Hence rich and large countries tend to be more diversified. In the context of North-South trade, this motivates a theoretical focus where only one of the trading parties is completely specialised.

In the case with FPE, general functional forms may be used to examine the HOS model outcome; as illustrated by the still widely used "hat calculus" of Jones (1965) or the duality approach of Dixit and Norman (1980). As noted by the latter authors (ibid., 113) and even by Stolper and Samuelson (1941, 354), more specific functional forms are required to address wages and prices in cases with complete specialisation. In this paper, we generally use Cobb-Douglas functions in order to make the analysis tractable. A desirable property of Cobb-Douglas production functions is also that countries with extreme factor composition are less efficient producers; in this way we also capture the supply-side constraints of very poor countries (see below).

The Cobb-Douglas technology is symmetrical in the sense that countries with too little or too much capital are both worse off. While the intuition behind "too little capital" is straightforward, it is less clear what it means empirically to have "too much capital". In the analysis, we shall mainly focus on the case when labour-abundant countries become completely specialised. We refer to the country with the higher (lower) capital-labour ratio as the North (South). We use the words *capital* and *labour*; although the empirical content may e.g. be skilled and unskilled labour, respectively.

While our main focus is on complete specialisation, the comparison between such cases and the situation with FPE sheds light on "regime change" as well as how the trade-wage link differs between the two situations. Furthermore, we need the autarky case as a benchmark for welfare comparisons. The article therefore proceeds as follows: In Section 2, we set up the model and derive the closed-economy model solution as well as the equilibrium with international trade and FPE. With the Cobb-Douglas technology, solutions are stylised and simple. We also show that a country's welfare depends strongly on its factor composition; maximum welfare in autarky is obtained when factor endowments correspond to the requirements for optimality in production (a weighted average for the two sectors). Some of the autarky results apply directly to international trade with FPE.

In Section 3, we describe in more detail the conditions for FPE, and examine how international trade affects welfare as well as goods and factor prices in the case with complete specialisation in one country. Although explicit analytical solutions for wages cannot be found in this case, an almost complete analytical characterisation is possible by means of implicit differentiation and other techniques. To make presentation easier, some of the comparative static results are presented in an analytical appendix, and numerical simulation is used to illustrate the results.

If the North trades with a specialised South (with a lower K/L ratio), it remains true that the nominal as well as the real wage of labour in the North is reduced from the autarky level. It is however not true that this adverse impact is stronger, the lower is the K/L ratio in the South. As long as the K/L ratio of the specialised country is lower than what is optimal in production, the Stolper-Samuelson relationship is reversed. In this range, a higher K/L ratio in the South will *reduce* the w/r ratio in the North. The threat to labour in the North is largest

when the South has an intermediate capital-labour ratio, because production in the South is then most efficient. What is worst for labour in the North, is however best for aggregate welfare. The South also gains from trade, and the more so, the more extreme is its factor composition. Countries with low productivity due to a very low K/L ratio can gain more from capital accumulation than from trade, but they also gain more from trade than others.

In section 3, we also show that if the South becomes larger, it is more likely that the North becomes specialised. For the North, this "China syndrome" will nevertheless be good for welfare. Finally, section 4 concludes by discussing empirical relevance as well as implications for research and policy. By taking into account cross-country differences in factor price *ratios* as well as *levels*, the model provides new hypotheses on the trade-wage link as well as the international income distribution.

2. The model, the closed economy, and international trade with factor price equalisation

2.1. The model

Consider, first, a single economy (also dropping country subscripts) with factor endowments K (with rewards r) and L (with rewards w); and two sectors A and B, using K_A, K_B, L_A, L_B in production. Factors can move freely between sectors. Production functions are Cobb-Douglas:

$$(1)\quad F_A = K_A^{\alpha} L_A^{1-\alpha} \qquad F_B = K_B^{\beta} L_B^{1-\beta} \qquad 0<\alpha<1,\ 0<\beta<1$$

With the Cobb-Douglas technology, factor cost shares are constant, e.g. for sector A

$$(2)\quad \frac{r K_A}{w L_A} = \frac{\alpha}{1-\alpha}$$

Hence firms will choose a factor input ratio K_A/L_A that maximises profits, and this depends on the factor price ratio. Observe, however, that the production functions (1) also define factor input ratios in each sector that are optimal in terms of physical production. For sector A, the factor ratio that makes the marginal products of the two factors equal, and is the most efficient one in physical terms, is $K_A/L_A=\alpha/(1-\alpha)$. [3] If the capital-labour ratio in production is different from this, reducing the physical amount of one factor and increasing the amount of the other factor can increase physical output.

This property of the Cobb-Douglas technology has two important implications for the analysis. First; there is a trade-off between domestic inequality and efficiency: When w/r=1, efficiency in production is larger.

[3] Observe that units for K and L cannot be chosen arbitrarily. In order to optimise physical productivity; should the marginal productivity of one unit of capital be equal to the marginal product of, say, one hour of work, or one year of work? We assume that this is technologically determined, and choose units so that with w/r=1, firms choose the factor input ratio that is also technically optimal. Since the same factor prices apply in both sectors, this rescaling also works for the other sector. Hence we measure factors in comparable "efficiency units".

Second; for an economy that spends all its resources in one sector, efficiency is greatest when its factor composition matches the technology in the sector; e.g. a country specialised in sector A is most efficient when its factor endowment ratio is equal to $\alpha/(1-\alpha)$. This property of the Cobb-Douglas technology is useful because it captures "supply-side constraints" in poor countries. If the South has too little capital, its output will be smaller.

With perfect competition, unit cost has to equal the price of each good. Using the *unit cost functions* for the Cobb-Douglas technology, this implies

(3a) $C_A/ F_A = Z_A r^{\alpha} w^{1-\alpha} = p_A$ where $Z_A = \alpha^{-\alpha}(1-\alpha)^{(\alpha-1)}$
(3b) $C_B/ F_B = Z_B r^{\beta} w^{1-\beta} = p_B$ where $Z_B = \beta^{-\beta}(1-\beta)^{(\beta-1)}$.

where C_A and C_B are total costs in each sector.

These relationships only apply when production is positive. If the country is diversified (produces both goods), we can divide (3a) by (3b) to obtain the Stolper-Samuelson relationship

$$(4) \qquad \frac{w}{r} = \left(\frac{Z_A}{Z_B}\right)^{\frac{1}{\alpha-\beta}} \left(\frac{p_A}{p_B}\right)^{\frac{1}{\beta-\alpha}}$$

The elasticity $1/(\beta-\alpha)$ is larger than one in absolute value, which illustrates the *magnification effect* related to SST: A change in the goods price ratio corresponds to a more than proportionate change in the factor price ratio.[4]

We assume that factors are fully utilised:

(5a) $K_A + K_B = K$ (5b) $L_A + L_B = L$

For a given factor price ratio and provided that both goods are produced, production in sector A will be[5] [6]

$$(6) \qquad F_A = \frac{1}{(\alpha-\beta)Z_A} \left(\frac{w}{r}\right)^{\alpha-1} \left\{K\,(1-\beta) - L\,\beta\,\frac{w}{r}\right\}$$

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[4] We deliberately say "corresponds to", rather than e.g. "leads to", since factor prices and goods prices are both endogenous in the model and it is not so clear that there is causality from one to the other. In a small open economy model where goods prices are exogenous, such causality is present. In almost any textbook in the field, however, the standard wording is "leads to" – also when the 2x2x2 HOS model is discussed (see, for example, Bhagwati et al. 1998, 62).

[5] The cost shares of K in sectors A and B are α and β, respectively, e.g. $rK_A/C_A=\alpha$ which gives $K_A=\alpha C_A/r$. Similarly, we have $K_B=\beta C_B/r$ and corresponding expressions for L_A and L_B. Substituting this into (5) and then using the unit cost functions (3) to substitute for C_A and C_B, we obtain the following two equations that can be solved for F_A and F_B:
$\alpha Z_A F_A (r/w)^{\alpha-1} + \beta Z_B F_B (r/w)^{\beta-1} = K$ and $(1-\alpha) Z_A F_A (r/w)^{\alpha} + (1-\beta) Z_B F_B (r/w)^{\beta} = L$

[6] The Rybczynski effect follows directly from (6) if we hold w/r constant and examine how production is affected by changes in K and L. If e.g. sector A is K-intensive ($\alpha>\beta$), an increase in K will lead to an increase in F_A, and a reduction in F_B. This result holds, however, only under the special assumption that prices are held constant, and is not a core a part of the 2x2x2 HOS model with flexible prices.

and a similar expression obtains for (with α and β interchanged, and B instead of A). Production is positive for both sectors only for a specified range of w/r. By setting (6) and F_B equal to zero, we find

(7a) $F_A=0$ if $w/r = K/L * (1-\beta)/\beta$ (7b) $F_B=0$ if $w/r = K/L * (1-\alpha)/\alpha$

If the factor price ratio falls outside the range defined by these two values, the economy will specialise in one of the two sectors. In the closed economy, the equilibrium factor prices are always within this range. With international trade, this needs not be the case.

In order to derive goods and factor prices, we need to introduce the demand side of the model. Still keeping things as simple as possible, we use a Cobb-Douglas utility function:

(8) $U = A^a B^{1-a}$

where A and B are consumption levels for goods A and B, with a and (1-a) as the consumption shares. With this function, we must have constant cost shares:

(9) $$\frac{p_A A}{p_B B} = \frac{a}{1-a}$$

2.2. The closed economy

In the closed economy, consumption must equal production; hence $A=F_A$ and $B=F_B$. Substituting into (9) from (6), and replacing p_A/p_B with w/r using (4), we obtain the solution

(10) $$\frac{w}{r} = \frac{K}{L}\frac{\theta}{\gamma}$$

where $\gamma = a\alpha+(1-a)\beta$ is the mean cost share of capital, weighted by consumption shares, and $\theta = a(1-\alpha)+(1-a)(1-\beta)$ is the mean cost share of labour (observe that $\gamma + \theta = 1$). Hence the factor price ratio is proportional to the ratio between the mean cost shares for the two factors in the economy, and inversely related to the K/L ratio; an increase in K leads to a relative reduction in r. [7]

Using (10), the price ratio p_A/p_B follows from (4). Production levels are[8] [9]:

(11a) $F_A = K^{\alpha} L^{1-\alpha} \lambda_A$ where $\lambda_A = a Z_A^{-1} \gamma^{-\alpha} \theta^{\alpha-1}$
(11b) $F_B = K^{\beta} L^{1-\beta} \lambda_B$ where $\lambda_B = (1-a) Z_B^{-1} \gamma^{-\beta} \theta^{\beta-1}$

[7] Some of these results are also derived by Deardorff (2000).

[8] Factor use in the two sectors will be $K_A = \frac{K\alpha a}{\gamma}$, $K_B = \frac{K\beta(1-a)}{\gamma}$, $L_A = \frac{L(1-\alpha)a}{\theta}$ and $L_B = \frac{L(1-\beta)(1-a)}{\theta}$.

[9] Observe that the Rybczynski Theorem no longer applies: The elasticities of production with respect to factor endowment changes (equal to the factor cost shares) are all positive and smaller than one. There is no "magnification effect", and no reduction in production (see also Jones (1965, 40)).

Utility in the closed economy will be equal to

$$(12) \quad U = K^{\gamma} L^{\theta} \lambda_A^{\ a} \lambda_B^{\ 1-a} = K^{\gamma} L^{\theta} \Delta Z_a^{\ -a} Z_B^{\ a-1} \gamma^{-\gamma} \theta^{-\theta}$$

where $\Delta = a^a (1-a)^{(1-a)}$.

In order to examine how a country's welfare is affected by its productivity, it is useful to examine the impact of factor proportions independently from country size. With an appropriate choice of units for K and L (see footnote 3), we may denote as R=K+L the total factor stock, and k and 1-k as the share of K and L, respectively. A change in R implies a change in country size for constant K/L, whereas a change in k implies that K/L is changed for constant R. To some extent, this corresponds to empirically observable phenomena; e.g. when unskilled is turned into skilled labour by means of education. In general, factor growth implies that k and R change simultaneously, but for analytical purposes it is useful to separate the two effects.

Using this notation in (12), we obtain

$$(12a) \quad U = R\, k^{\gamma} (1-k)^{\theta} \lambda_A^{\ a} \lambda_B^{\ 1-a}$$

An important property of the HOS model that has received little focus in the international trade literature, is that the welfare level of a country depends strongly on its factor endowment ratio. It is straightforward to show that:

- Production in sector A is maximized when $k=\alpha$.
- Production in sector B is maximized when $k=\beta$.
- Welfare is maximized when $k=\gamma$.

The last property is confirmed by taking the first- and second-order derivatives of (12a) with respect to k, and the first two propositions are confirmed by using a similar technique on (11a,b) (also using "factor share notation" for K and L).

Hence *autarky welfare reaches a maximum when $k=\gamma$ or equivalently $K/L=\gamma/\theta$*, since average productivity is then highest. The better the match between factor endowments and the mean cost shares in production, the more efficient will the economy be. This property is also important in the context of international trade: Even if countries can gain from trade, they can sometimes gain more by improving their factor composition. Observe also that with international trade, the welfare maximum may be obtained for other values of k, as will be shown in the following.

The property that welfare is highest when a country has the "optimal factor composition" resembles some results from the growth literature. Barro and Sala-i-Martin (1995, 174) e.g. obtain a similar steady-state K/L ratio ($=\alpha/(1-\alpha)$) in a one-sector growth model.

2.3. International trade with factor price equalisation

Consider that two economies with different K/L ratios are integrated, with free trade between them for both products. We assume that factors are immobile between countries. We use the notation $K_1=k_1R_1$, $K_2=k_2R_2$, $L_1=(1-k_1)R_1$, $L_2=(1-k_2)R_2$ for the factor stocks in the two countries, with $K_W=K_1+K_2$, $L_W=L_1+L_2$ as "world" factor stocks.

With free trade between the two countries, goods prices will be equalized. If both countries remain diversified, the Stolper-Samuelson relationship (4) must apply, and factor prices will be equalized in the two countries. Denoting the common world factor price ratio w*/r*, the diversification range is defined by (7). For example, country 1 specialises in A when $F_{1B}=0$, and if sector A is L-intensive, this implies:

(13) $F_{1B}=0$ if $w^*/r^* \geq K_1/L_1 * (1-\alpha)/\alpha$

If sector A is K-intensive, the inequality sign is reversed. Similar conditions apply specialization in sector B, and for country 2.

If both countries remain diversified, the solution (10) applies to the world economy, i.e.

$$(14) \qquad \frac{w^*}{r^*} = \frac{K_W}{L_W}\frac{\theta}{\gamma}$$

which will be between the autarky factor price ratios in countries 1 or 2, since K_W/L_W must lie between the K/L ratios of the two countries.

Given the solution (14), other properties of the case with FPE can be derived. These are well known from the literature and need not be replicated here.

3. International trade with complete specialisation

3.1. Possible patterns of specialisation

If e.g. country 1 is specialised in sector A, and we still assume full employment, all its factor endowments are used in that sector. Production of the good is then $F_{1A}=K_1^{\alpha}L_1^{1-\alpha}$. This non-responsiveness of production implies that prices have to bear the burden of adjustment, and terms-of-trade effects become more important. Changes in the price ratio p_A/p_B affect the *level* of factor prices in the completely specialised country, but not the factor price ratio, which has to obey (2), or with appropriate subscripts

$$(15) \qquad \frac{w_1}{r_1} = \frac{1-\alpha}{\alpha}\frac{K_1}{L_1}$$

Hence for a specialised country, the factor price ratio will be determined solely by the domestic K/L ratio and the technology in production. Changes in K_2 or L_2 or β or the consumption share a will have no impact on w_1/r_1, but affect relative goods prices and thereby the *levels* of w_1 and r_1.

By setting (14) equal to (15), we define the border case between FPE and specialisation with $F_{B1}=0$. Defining similar conditions for $F_{A1}=0$, $F_{B2}=0$ and $F_{A2}=0$, we delineate the FPE set in the model. Figure 2 illustrates possible cases of specialisation, based on numerical simulation with equal country size ($R_1=R_2$), $K_W=L_W$, equal consumption shares (a=0.5), and technology parameters $\alpha=1/3$ and $\beta=2/3$ (hence sector A is L-intensive).

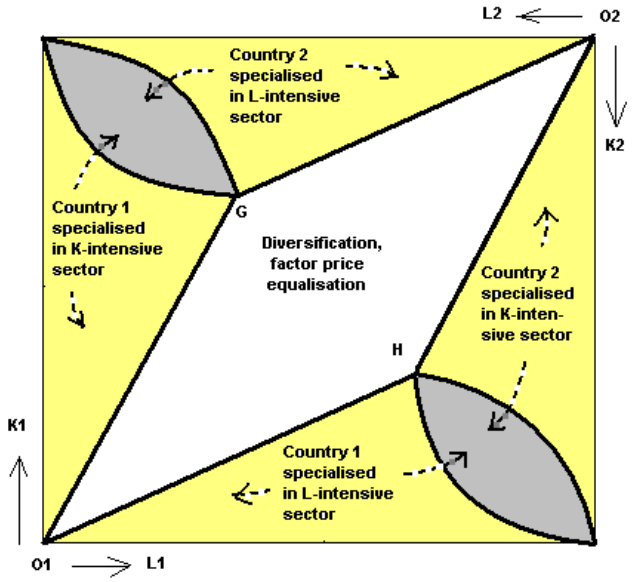

Fig. 2: Cases with complete specialisation

The ray O_1H defining when country 1 is specialised in sector A is e.g. defined by $K^*_{1A} = \frac{\overline{K}_W}{\overline{L}_W}\frac{\theta}{\gamma}\frac{\alpha}{1-\alpha}L^*_{1A}$, where the bars indicate that world factor stocks are considered constant in this calculation. In the simulated case, the rays O_1H and O_1G will have slopes 0.5 and 2, respectively. The conditions for complete specialisation in both countries are more complex, and the shape of the corner areas in the Figure is drawn based on numerical simulation, using later results.[10]

The share of the FPE set in the total factor space area will be equal to

$$(16) \qquad Share_{FPE} = \frac{a(1-a)|\alpha-\beta|}{\gamma(1-\gamma)}$$

Hence the more similar the two sectors are, the less likely is FPE. In the case simulated above, where a=γ and β-α=1/3, the FPE parallelogram covers 1/3 of the factor space. Observe also that the size of world factor stocks does not enter in the expression; so e.g. doubling the world stock of L will not change the "probability" for FPE.[11]

According to Figure 2, non-FPE is an outcome at least as likely as FPE, so there is good reason to study cases with complete specialisation. There are six possible cases of complete specialisation, with both countries specialised in two of them, and one country specialised in the remaining four. Figure 2 indicates that the outcome with complete specialisation in only one country is more likely. This is one reason why we shall mainly focus on this case.

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[10] See also Markusen and Venables (2000, 217) for a similar picture in a model with monopolistic competition in one of the sectors. These authors also show that when trade costs are added to the model, the FPE set shrinks to a line.

[11] Since expressions of the type x(1-x) reach their maximum for x=0.5, we also see that FPE is more likely if technologies are "skewed" towards K or L (γ different from 0.5), and more likely if consumption of the two goods is balanced (parameter *a* close to 0.5).

Another reason is that complete specialisation in both countries creates an extreme model where the income share of a country is independent of its size and determined by the utility function (8). The demand shares for each good follow from this, and thereby also each country's income share. So if country 1 produces good A and country 2 good B, the income shares must be $Y_1/Y_2=a/(1-a)$. The cost shares in production then determine the allocation of income between factors in each country. Hence demand determines income distribution across countries, and technology and factor stocks determine income distribution within countries. In this setting, it is good to be a small country, since demand from the larger partner will raise income. Given that factor substitution is not possible, adjustment only occurs via price changes. Analytically, the case with complete specialisation in both countries is the easier one. In the Appendix, Section A3, the properties of this case are derived (see also Deardorff 2001b).

With complete specialisation in only one country, the presence of diversification in the other country implies that adjustment occurs through factor substitution as well as prices. Therefore, this is an intermediate case between the standard HOS model and the case with complete specialisation in both countries.

3.2. Complete specialisation in one country

In order to derive the analytical properties of the case when only one country is completely specialised, we assume (without loss of generality) that country 1 is specialised in A and country 2 diversified. The condition for this to occur is given by (13). The factor price ratio in country 1 is then (15), and its production is $F_{1A} = K_1^{\alpha}L_1^{1-\alpha}$. When analysing how k_1 affects the model outcome when country 1 is specialised, we shall use the term k_{1A} for the (upper or lower) value of k_1 that leads to specialisation in sector A, and k_{1B} for the value of k_1 that leads to specialisation in sector B.

The value of k_{1A} is obtained by setting (14)=(15), which gives a quadratic equation that can be solved for k_1.[12] The comparative static results for k_{1A} are most easily derived using implicit differentiation. If sector A is L-intensive, we obtain

$$(17)\quad \frac{\partial k_{1A}}{\partial k_2} > 0 \qquad \frac{\partial k_{1A}}{\partial R_2} > 0 \qquad \frac{\partial k_{1A}}{\partial R_1} < 0$$

If sector A is K-intensive, the signs are all reversed.

If sector A is L-intensive, specialisation must occur if $0<k_1\leq k_{1A}$. The higher is k_{1A}, the wider is this range. The larger is country 2, the higher is k_2, and the smaller is country 1; the more likely is complete specialisation. A small country will by definition be more different from the world average than the large one, and therefore more likely becomes specialised.[13] The empirical evidence presented in the introduction supports this prediction.

[12] In this case, it is not plausible to assume that world factor stocks are constant, since changes in the factor endowments of one country would then be equivalent to factor movements, which have been ruled out by assumption.

[13] It is sometimes stated that the condition for specialisation is that the difference in factor endowment ratios must be smaller than the difference in sectoral factor intensities (see e.g. Deardorff (1994)). As seen from (17), country size differences are also important.

Since country 1 only produces A, its total income (Y_1) must be $p_A F_{1A}$. Inserting this into the demand function $A_1=ap_A^{-1}Y_1$, consumption of A will be $A_1 = aF_{1A}$. Hence country 1 has to export $F_{1A}-A_1=(1-a)\,F_{1A}$. This has to equal the difference between production and consumption of good A in country 2:

(18) $(1-a)\,F_{1A} = A_2 - F_{2A}$

Production of good A in country 2 is given by (6) (with appropriate subscripts). Consumption of A in country 2 is given by the demand function $A_2=ap_A^{-1}Y_2$, where Y_2 is income. Now $Y_2 = r_2K_2 + w_2L_2 = r_2\,[K_2 + L_2\,(w_2/r_2)]$, and from unit cost = price, equation (3), we obtain $r_2/p_A = Z_A^{-1}\,(w_2/r_2)^{\alpha-1}$. Inserting this into the demand function gives $A_2 = a\,Z_A^{-1}\,(w_2/r_2)^{\alpha-1}\,[K_2 + L_2\,(w_2/r_2)]$. Using these results and simplifying, equation (18) can be expressed as

(19) $$\Phi_A = \frac{1}{(\beta-\alpha)Z_A}\left(\frac{w_2}{r_2}\right)^{\alpha-1}\left(\theta K_2 - \gamma L_2\frac{w_2}{r_2}\right) - (1-a)K_1^{\alpha}L_1^{1-\alpha} = 0$$

where the factor price ratio w_2/r_2 is the only unknown.[14] Hence (19) determines the factor price ratio. While this non-linear equation is not easily solved, we can examine the outcome by means of implicit differentiation.

While (19) implicitly determines w_2/r_2, it is not sufficient to determine fully the factor prices in the model. First, the nominal value of all prices is undetermined. In order to solve this, we use (without loss of generality) good B as the numeraire, $p_B=1$. Hence we express all prices in terms of one unit of B. Given the B is produced only in country 2, the price=unit cost equation (3) implies:

(20) $$p_B = Z_B\,r_2^{\beta}\,w_2^{1-\beta} = 1$$

With w_2/r_2 determined by (19), equation (20) determines the levels of w_2 and r_2.

With the factor price ratio in country 1 given by (15), the last missing link is the *level* of factor prices in country 1. Since both countries produce good A, the unit cost equation (3) implies that

(21) $$p_A = Z_A\,r_1^{\alpha}\,w_1^{1-\alpha} = Z_A\,r_2^{\alpha}\,w_2^{1-\alpha} \qquad \text{or} \qquad r_1^{\alpha}\,w_1^{1-\alpha} = r_2^{\alpha}\,w_2^{1-\alpha}$$

Equations (15) and (19-21) then implicitly determine the levels for all factor prices, and p_A also follows from (21). Using these equations, we find the comparative static results summarised in Table 1. The analytical derivation of the results is found in the Appendix, Section A1.[15] Observe that the results are only valid within the range of factor endowment combinations that are

[14] In case country 1 is specialised in sector B, the equilibrium condition is $\Phi_B = \frac{1}{(\alpha-\beta)Z_B}\left(\frac{w_2}{r_2}\right)^{\beta-1}\left(\theta K_2 - \gamma L_2\frac{w_2}{r_2}\right) - aK_1^{\beta}L_1^{1-\beta} = 0$. This is used for simulations underlying various Figures.

[15] In this Appendix, comparative static results using K_1, L_1 instead of R_1, k_1 etc. are also provided.

consistent with this pattern of specialisation. In the table, we also show changes in production in the specialised country, since this is important for the results.

Table 1: The impact of factor endowment changes on factor prices and goods prices when country 1 is specialised in sector A.
Shaded areas: International effects.
Signs in brackets: Suggested by numerical simulation, not confirmed analytically

Endogenous variable ↓	$\alpha<\beta$ (sector A is L-intensive)					$\alpha>\beta$ (sector A is K-intensive)				
	R_1	k_1		R_2	k_2	R_1	k_1		R_2	k_2
		$k_1<\alpha$	$k_1>\alpha$				$k_1<\alpha$	$k_1>\alpha$		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
w_1	÷	(+)	+	+	+	÷	(+)	+	+	÷
r_1	÷	÷	(÷)	+	+	÷	÷	(÷)	+	÷
w_1/r_1	0	+	+	0	0	0	+	+	0	0
w_2	÷	÷	+	+	+	+	+	÷	÷	+
r_2	+	+	÷	÷	÷	÷	÷	+	+	÷
w_2/r_2	÷	÷	+	+	+	+	+	÷	÷	+
p_A	÷	÷	+	+	+	÷	÷	+	+	÷
F_{1A}	+	+	÷	0	0	+	+	÷	0	0

Note: Analytical verification of the results is provided in the Appendix.

Due to the symmetry of the model, comments focus on the case when A is labour-intensive; i.e. columns (1)-(5); i.e. with the South completely specialised.

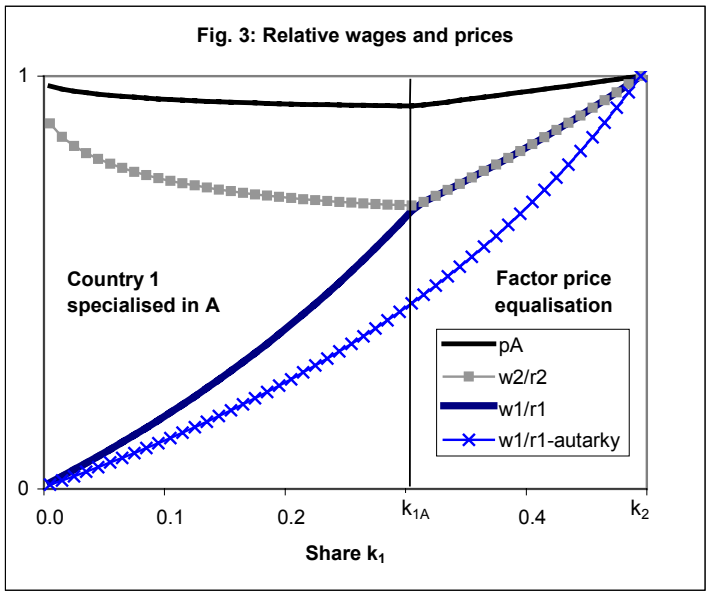
A general message from Table 1 is that the domestic impact of factor endowment changes is partly similar to the standard HOS model with FPE, but the cross-border impact is fundamentally different. For example, changes in the factor endowment composition in the North (k_2) have a standard domestic impact (higher k_2 lowers r_2). But the factor price ratio in the specialised South is unaffected by these changes, so cross-border effects only arise due to changes in the terms-of-trade that affect the factor price level in the South. Since the South is specialised in the L-intensive sector, an increase in k_2 will change terms-of-trade in favour of the South and increase its factor price levels.

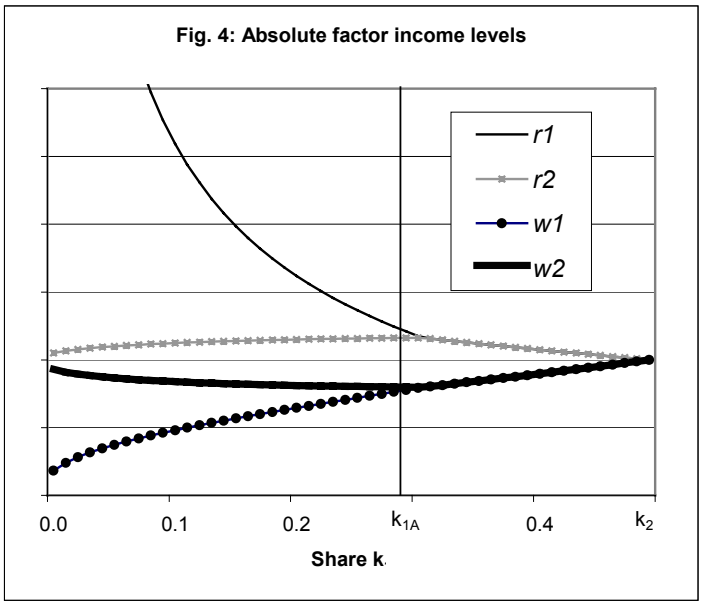
If the North grows in size (increased R_2), it will increase the demand for imports from the South and have a similar positive impact on factor price levels. Size growth in the South will have the opposite effect, by increasing production and thereby lowering p_A. While small countries are more likely to become specialised, they will benefit from sales to the larger market.

As seen from Table 1, the impact of changes in the South's factor endowment composition (k_1) is ambiguous and depends on whether k_1 is smaller or larger than α. The reason is that production efficiency in the specialised South is highest when $k_1=\alpha$, so increases in k_1 beyond this reduce efficiency in the South. It depends on the size and factor composition in the North whether we can have $k_{1A}>\alpha$; this will not necessarily be the case. If $\alpha<\beta$ and the North has a factor endowment ratio corresponding to what is optimal for welfare in autarky, i.e. $k_2=\gamma$, we can never have $k_{1A}>\alpha$. If the capital share in the North is larger than γ, however, this is possible, and it is more likely if the North is large. We shall therefore illustrate two cases, one in which column (3) in Table 1 does not apply, and another case when it does.

3.2.1. Integration with an "optimal" North

With a factor endowment ratio $k_2 = \gamma$ in the North, and equal country size ($R_1=R_2$), we will never have $k_{1A}>\alpha$. Hence for the range when the South is specialised, only column (2) in Table 1 applies. Figure 3 shows factor price ratios, and Figure 4 factor price levels, in this case.





Within the FPE range, factor price ratios and levels in the two countries coincide. Increases in k_1 lead to a higher w/r ratio, falling r and increasing w in both countries. With specialisation in the South, however, the factor endowment-wage link is reversed: In this case, increases in the capital share in the South improve its productivity, raise output, lower the relative price of A and hurt labour in the North. Hence changes in production efficiency, and corresponding changes in the terms-of-trade, create a reversal of the

international trade-factor price link compared to the standard case with FPE. The curve for p_A in Figure 3 illustrates the terms-of-trade effects. For good A, the relative price reaches a minimum at the "specialisation point" when $k_1=k_{1A}$.

In the South, the domestic impact of changes in the capital share k_1 is mainly similar to the standard HOS case; an increase in k_1 leads to an absolute and relative fall in r_1, and higher w_1.[16] Even if the gain for labour in the South is dampened by a falling relative price for good A, a relative increase in the supply of capital, together with productivity growth, implies that labour in the South is clearly better off.

Turning to welfare, Table 2 summarises analytical results concerning the impact of factor endowment changes on welfare. In the Appendix, Section A2, these results are derived. In Table 2, two welfare measures are provided; welfare gains from trade $W_i=U_i/U_{i\text{-autarky}}$, and absolute welfare levels U_i (equation (8)). Observe that W_i is a relative measure. The absolute welfare levels U_i depend on country size, and derivatives with respect to own country size are therefore not reported (shaded cells). There are a few cases where clear-cut analytical results could not be obtained, and for some of these, we include expected signs based on numerical simulation (entries in brackets).

Table 2: The impact of factor endowment changes on welfare when country 1 is specialised in sector A.

Signs in brackets: Suggested by numerical simulation, not confirmed analytically

Endogenous variable ↓	$\alpha<\beta$ (sector A is L-intensive)					$\alpha>\beta$ (sector A is K-intensive)				
	R_1	k_1		R_2	k_2	R_1	k_1		R_2	k_2
		$k_1<\alpha$	$k_1>\alpha$				$k_1<\alpha$	$k_1>\alpha$		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
W_1	÷	÷	(÷)	+	+	÷	(+)	+	+	÷
W_2	+	+	÷	÷	?	÷	+	÷	+	?
U_1		(+)	(÷)	+	+		(+)	(÷)	+	÷
U_2	+	+	÷		?	+	+	÷		?

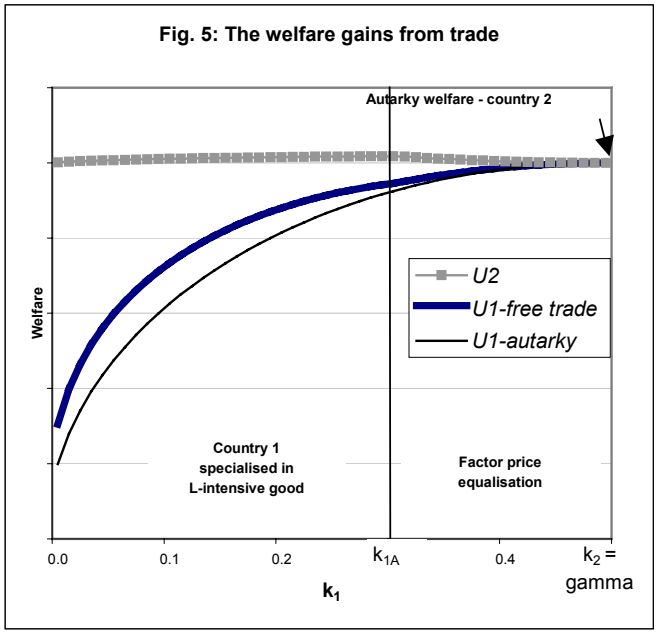
Note: Analytical verification of the results is provided in the Appendix, Section A2.

The results show that:

- The *welfare gains from trade increase in partner country size and decrease in a country's own size. Welfare levels also increase with partner country size*.
- Since the model also applies for country size equal to zero, this also shows that *both countries gain from trade* (when partner country size increases from zero).
- *For the specialised country, the welfare gains from trade are larger, the more extreme is the country's factor composition* (see columns (2) and (8)).
- Together with numerical simulations, the results suggest that when columns (3) and (7) apply, both countries obtain a maximum welfare level for $k_1=\alpha$., and the diversified country obtains the largest gains from trade at this value. This is, however, not fully confirmed analytically.

Figure 5 shows the welfare levels in the case illustrated above, together with autarky welfare levels so that the welfare gains from trade can also be seen.

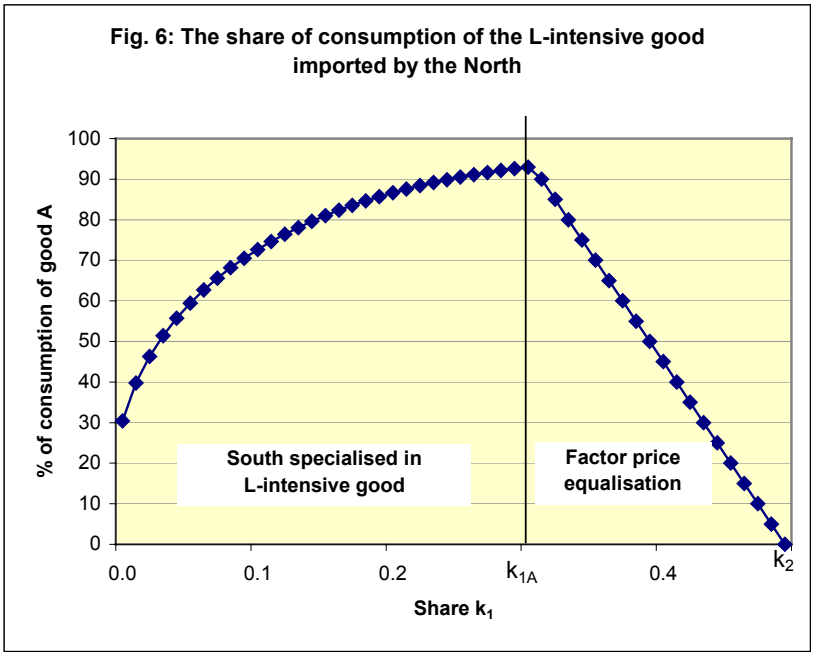
[16] These level effects are only partially supported analytically (see Table 1), but simulations suggest that this statement holds generally.



The lower curve in the Figure shows autarky welfare for the South. Observe how strongly this depends on the endowment composition. As shown in Section 2.2, the maximum is obtained for $k_1=\gamma$. For the North, with the optimal capital share $k_2=\gamma$ (in this case 0.5), autarky welfare is equal to the maximum for country 1.

The two upper curves show that both countries gain from trade. Within the FPE set, the gain for both countries is larger; the more different are their factor endowment ratios. With complete specialisation, however, this only holds for the specialised South. For the South, the relative improvement from autarky to free trade is larger, the more extreme is its factor composition. For the diversified North, however, trade in the range with specialisation in the South leads to smaller benefits as the factor composition in the South becomes more extreme. In the whole range with specialisation in the South, however, the North is better off with trade than autarky. Hence with complete specialisation, both countries gain from trade.

For the diversified North, what is good for welfare is bad for labour: The welfare gain is larger, the lower is the import price, but then the negative impact on Northern wages is also largest. Countries in the South with intermediate K/L ratios therefore represent the strongest threat to labour in the North. But if the K/L ratio in the specialised South becomes sufficiently high, FPE will set in, and the SST will again apply to the world economy. When the threat of labour-intensive imports into the North is at the greatest, it suddenly disappears. This is shown in Figure 6.



Hence when the "danger" of import competition is at its largest, imports suddenly decline if the K/L ratio in the South increases further.

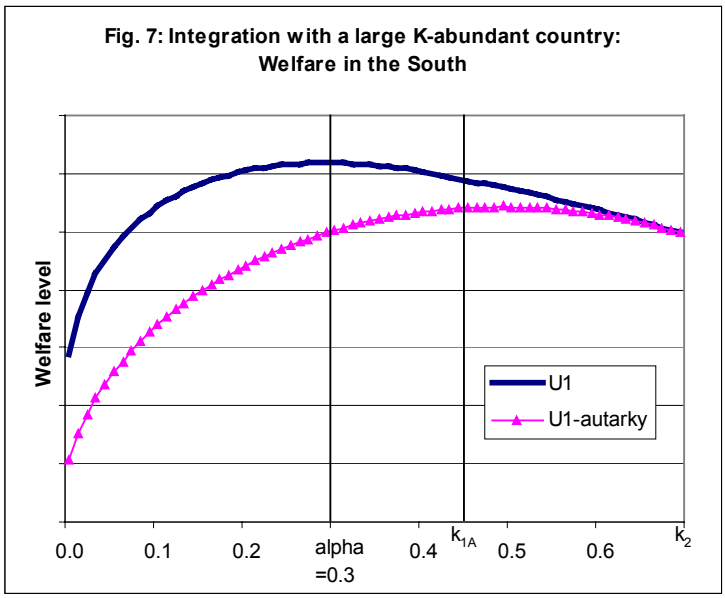
Due to the "break point" at k_{1A}, the HOS model can shed light on "stages of growth", if complete specialisation is accounted for. This is known from the recent literature on multi-good HOS models where countries can move between different "cones of diversification" (see, for example, Deardorff (2000), Schott (2003)). What the analysis above shows, is that there is a corresponding "break point" for the international transmission from trade to goods and factor prices, and that effects of this kind occur even in the 2x2x2 HOS model.

3.2.2. A theory of the "twin peaks" income distribution?

If the specialisation point $k_{1A}>\alpha$, Table 1, column (3) shows that a qualitatively different "stage of development" is added to the model. In the range when $\alpha<k_1<k_{1A}$, an increase in k_1 will make the specialised South a less efficient producer. Increases in k_1 then increase the relative price p_A, labour in the North is better off, and there is again a trade-wage link similar to the standard case with FPE.

We illustrate this case numerically by increasing the size of country 2 ($R_2=5R_1$) as well as its capital share ($k_2=\beta$). Figure 7 shows welfare in the specialised country in this case, for the range when $k_1\leq k_2$.[17]

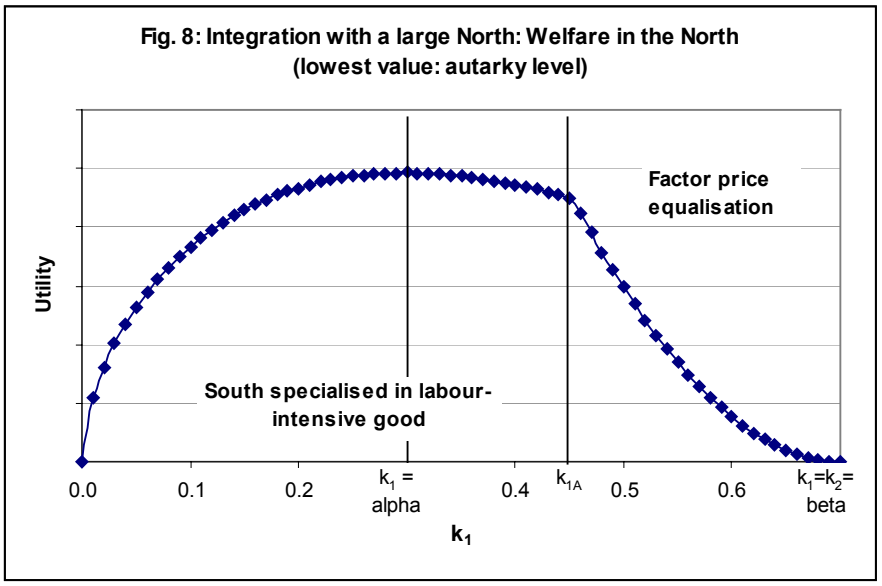
[17] Other parameter values: α=o.3, β=0.7, a=0.5.



Hence when $\alpha<k_1< k_{1A}$, corresponding to column (3) in Table 1, welfare in the South is inversely related to k_1. Since productivity in the South is highest when $k_1=\alpha$, further increases in its capital share is bad for productivity and welfare in the South. In this case, the South is better off with complete specialisation than with FPE. Hence complete specialisation is not synonymous with poverty. Since the North is large, the market for Southern exports is large, and this adds to the welfare gain.

Since nominal income mainly follows welfare, an implication is that the relationship between factor endowment ratios and income levels is not monotonous. The result may also be relevant for studies of the world income distribution: The evidence suggests that this is bimodal or "twin-peaked", with many poor and a number of rich countries, but few in the middle (Quah 1996). If the mechanism shown above is embedded in a growth framework, it is likely that such polarisation would occur.

In the illustrated case, the North is much larger, and its welfare level is therefore little affected by changes in the factor composition of the South. The qualitative impact is nevertheless of interest, and this is shown in Figure 8:



With complete specialisation in the South, the capital-rich North is always better off than in autarky, and for a wide range, welfare is higher than what can be obtained in the FPE range. A maximum is obtained when the South is most efficient; with $k_1=\alpha$.

The analysis above may give the false impression that only labour-abundant countries can become specialised. This is, however, not necessarily the case: If the South becomes large enough, it is the North that becomes specialised, and this may happen even for intermediate K/L ratios. In recent years, international trade has changed dramatically due to growth in Asia, and due to the integration of large and poor countries such as China into the world economy. What will happen in the model if the South grows larger?

In order to illustrate the possibility that the North becomes completely specialised, we can mirror the former case, and just assume that the large country 2 has a capital share equal to α instead of β (still assuming $\alpha<\beta$). Now the North would be specialised in the capital-intensive good B, and Northern welfare would be a mirror image of Figure 7. In Figure 7, the specialised South reaches a welfare maximum for $k_1= \alpha$. With the "China syndrome", the specialised North reaches a welfare maximum for $k_1=\beta$.[18] Hence in both cases, welfare with complete specialisation is higher than in the FPE range. In the case illustrated in section 3.2.1, this is not the case.

4. Implications and empirical relevance

The analysis has shown that when the HOS model is extended beyond the FPE set, several important modifications occur. In particular, the monotonous link between factor endowments and factor prices is interrupted, and the Stolper-Samuelson relationship is modified. Non-FPE allows terms-of-trade effects to play a larger role, and the analysis suggests that completely specialised countries

[18] It is straightforward to show analytically that this conclusion holds when $R_2\rightarrow\infty$. The numerical simulations suggest that the conclusion also holds generally, but we have not been able to prove this analytically.

can gain more from trade than the diversified ones. Complete specialisation is not synonymous with poverty; in some situations countries are better off with complete specialisation than with FPE. For trade in general and North-South trade in particular, complete specialization should be considered as a real possibility rather than a special case.

To what extent does this model capture empirical phenomena? Some indications to this effect were provided in the introduction. By allowing factor price levels as well as ratios to differ across countries, the model is also more realistic than the standard case with FPE in terms of explaining international income differences. The empirical literature suggests that factor composition may to some extent explain the international income distribution. While the recent consensus has been that technology differences are more important than differences in factor endowments, Caselli (2004, 4) concludes "if the elasticity of substitution between physical and human capital is low enough, observed differences in factor endowments become able to explain the bulk of the income distribution". The analysis of complete specialisation adds new hypotheses to empirical research on these issues.

With respect to inequality within countries, the model is partly in line with the standard model, and partly different. Along with the standard model, the model predicts that trade will lead to greater inequality in the North, and less inequality in the South. The negative correlation between import prices and wages in the North also holds irrespective of whether there is FPE or complete specialisation in the South. While greater inequality in the North is mainly confirmed empirically, the evidence for the South is more mixed and partly contrary to the hypothesis (see e.g. S. Davis (1992) or D.R. Davis (1996)). The model presented here does not present a clear solution to this last puzzle, except one hypothesis that might be relevant: For completely specialised countries, changes in the volume of trade should not affect the ratio but only the level of factor prices. Hence analysis of the trade-wage link should not only take into account relative wages, but also wage levels. This also applies to the North; to the extent that integration with large developing countries leads to complete specialisation.

For empirical research on the trade-wage link in the North, the analysis has predictions that differ from the standard HOS model in some important respects: The monotonous relationship between K/L ratios in the South and the trade-wage impact, as well as import prices and import volumes, are broken with complete specialisation. For studies of the "factor content of trade", this has obvious implications.

Our conclusions have been obtained in a static model, but nevertheless have implications for growth, by indicating that countries pass through stages of growth as they accumulate capital. The experience of fast structural change in emerging economies such as Hong Kong or Korea is a case in point. Such aspects have already been examined in a dynamic context (see e.g. Deardorff (2000, 2001) and Schott (2003)), and research can be further extended in this direction. As noted in Section 3.2.2, the model may potentially shed light on the "twin-peaked" world income distribution, if the mechanisms described here are built into growth models.

For policy, the analysis suggests that countries generally gain from trade, and that completely specialized countries may gain more than diversified ones. Even if poor countries suffer from supply-side constraints due to a limited

capital stock, they should engage in international trade. On the other hand, the analysis also shows that for the poorest countries, policies aimed at promoting accumulation of skills and physical capital may be even more important than free trade. For rich countries, a message is that integration with large poor countries can push them in the direction of complete specialization, but this can be an advantage for welfare. For labour in the North, trade with the poorest countries is least harmful, and this provides a possible political-economy explanation of why some rich countries provide more generous market access to the poorest developing countries.

The real world certainly contains more than two countries, and an issue is in what way our results apply in a multi-country setting. Examining factor price effects in models with many countries and different cones of diversification could shed more light on this. A possibility is that similar mechanisms apply to the relationship between "convergence clubs" rather than between individual countries.

The Cobb-Douglas version the HOS model is a stylized story that does not account for a number of empirically important real-world phenomena. Results and conclusions should be interpreted in this light. It should also be recalled that for most empirical phenomena related to international trade, there are alternative theoretical explanations. When, for example, large countries are more diversified, this could alternatively be caused by the "home market effects" of new trade theory, or by externalities related to technology. The mechanisms described here therefore only represent one part of the puzzle.

Appendix: Analytical supplement

A1. Factor price effects with specialisation in one country

This Section provides analytical support for the results presented in Table 1 in the main text, concerning the impact of factor endowment changes on factor and goods prices. Observe that the following results only apply within the range of factor endowment combinations that are consistent with the case studied: Country 2 is diversified and country 1 is specialised in the production of good A.

We reformulate (2) as

$$\text{(A1)} \qquad \frac{w_1}{r_1} = \frac{1-\alpha}{\alpha}\frac{k_1}{1-k_1}$$

R_1, k_2 and R_2 have no impact on w_1/r_1, and we obtain $\partial(w_1/r_1)/\partial k_1>0$. Next, write (19) with "factor share notation" as

$$\text{(A2)}\ \Phi_A=\frac{1}{(\beta-\alpha)Z_A}\left[\frac{w_2}{r_2}\right]^{\alpha-1}\left(\theta k_2-\gamma(1-k_2)\frac{w_2}{r_2}\right)R_2-(1-a)R_1k_1^{\alpha}(1-k_1)^{1-\alpha}=0$$

With w_2/r_2 as the only endogenous variable, the rules for implicit differentiation tell that $\partial y/\partial x=-(\partial\Phi/\partial x)/(\partial\Phi/\partial y)$, where y here is w_2/r_2, and x can be any other (exogenous) variable. The partial derivatives are:

$$\text{(A3)} \quad \frac{\partial \phi_A}{\partial w_2 / r_2} = \frac{R_2}{(\beta-\alpha)Z_A}\left[\frac{w_2}{r_2}\right]^{\alpha-2}\left((\alpha-1)\theta k_2 - \alpha\gamma(1-k_2)\frac{w_2}{r_2}\right)$$

The bracketed expression to the right is negative; hence the sign depends on $\beta-\alpha$.

$$\text{(A4)} \quad \frac{\partial \phi_A}{\partial R_2} = \frac{1}{(\beta-\alpha)Z_A}\left[\frac{w_2}{r_2}\right]^{\alpha-1}\left(\theta k_2 - \gamma(1-k_2)\frac{w_2}{r_2}\right)$$

In (A2) the term with k_1, R_1 is negative. Hence the first term in (A2) has to be positive, and for this reason (A4) is also positive.

$$\text{(A5)} \quad \frac{\partial \phi_A}{\partial k_2} = \frac{R_2}{(\beta-\alpha)Z_A}\left(\frac{w_2}{r_2}\right)^{\alpha-1}\left(\theta + \gamma\frac{w_2}{r_2}\right)$$

$$\text{(A6)} \quad \frac{\partial \phi_A}{\partial R_1} = -(1-a)k_1^{\alpha}(1-k_1)^{1-\alpha} < 0$$

$$\text{(A7)} \quad \frac{\partial \phi_A}{\partial k_1} = (1-a)R_1 k_1^{\alpha-1}(1-k_1)^{-\alpha}(k_1-\alpha)$$

The signs of the partial derivatives are thereby as follows:

Table A1: Signs of partial derivatives of ϕ_A

Partial derivative of ϕ_A with respect to:	$\alpha < \beta$	$\alpha > \beta$
w_2/r_2	÷	+
k_1	+ if $k_1 > \alpha$ ÷ if $k_1 < \alpha$	
R_1	÷	÷
k_2	+	÷
R_2	+	+

Using the implicit differentiation formula, the signs reported in Table 1 in the main text for w_2/r_2 can then be derived.

Next, given that $p_B=1$ and good B is produced in country 2, the unit cost expression (3) implies $Z_B r_2^{\beta} w_2^{1-\beta}=1$. This is equivalent to

$$\text{(A8)} \quad Z_B\, w_2 = (w_2/r_2)^{\beta} \qquad \text{or} \qquad Z_B\, r_2 = (w_2/r_2)^{\beta-1}$$

Differentiating on both sides with respect to any exogenous variable except for β (which is included in Z_B), observe that the impact of any such variable on w_2

must have the same sign as the impact on w_2/r_2, and the impact on r_2 must have the opposite sign.

Furthermore, the impact of factor endowment changes on p_A is also determined by the results for w_2/r_2. Given that country 2 is diversified, the Stolper-Samuelson relationship (4) must hold. With $p_B=1$ the appropriate version is

$$\text{(A9)} \quad \frac{w_2}{r_2} = \left[\frac{Z_A}{Z_B}\right]^{1/(\alpha-\beta)} p_A^{1/(\beta-\alpha)}$$

Hence if $\alpha>\beta$, p_A moves opposite to w_2/r_2, but with $\alpha<\beta$ the impact is in the same direction. This applies with respect to factor endowment changes as well as changes in the consumption shares a, but not necessarily with respect to changes in the technology parameters α and β.

The final step is to derive the impact of factor endowment changes on the absolute factor price levels in country 1. We have

$$p_A = z_A r_1^{\alpha} w_1^{1-\alpha} \qquad \text{or equivalently} \qquad p_A = z_A w_1 \left(\frac{w_1}{r_1}\right)^{-\alpha}$$

w_1/r_1 is unaffected by R_1, R_2 and k_2, so for changes in these three variables, p_A and w_1 must move in the same direction. In order to check the impact of changes in k_1, we differentiate on both sides of the last expression. This gives

$$\text{(A10)} \quad \frac{\partial p_A}{\partial k_1} = Z_A \left\{ \frac{\partial w_1}{\partial k_1}\left(\frac{w_1}{r_1}\right)^{-\alpha} - \alpha\, w_1 \left(\frac{w_1}{r_1}\right)^{-\alpha-1} \frac{\partial w_1/r_1}{\partial k_1} \right\}$$

Now if the derivatives of p_A and w_1/r_1 have the same signs, the sign of the derivative of w_1 must be similar. This is the case for $s_{k1}>\alpha$, both when $\alpha>\beta$ and when $\alpha<\beta$. If the derivatives of p_A and w_1/r_1 have opposite signs, however, we cannot determine the sign of $\partial w_1/\partial k_1$ in this way.

Using (A1), we can express w_1 in terms of r_1, and write p_A as

$$p_A = \frac{1}{\alpha} r_1 \left(\frac{k_1}{1-k_1}\right)^{-\alpha}$$

Again differentiating on both sides, we find that if $\partial p_A/\partial k_1$ is negative, then $\partial r_1/\partial k_1$ must also be negative. This is the case for $k_1<\alpha$, with $\alpha<\beta$ as well as $\alpha>\beta$. If $\partial p_A/\partial k_1$ is positive, we cannot determine the sign of $\partial r_1/\partial k_1$ in this way.

Hence the impact of changes in k_1 on the absolute factor price levels in country 1 can be unambiguously determined in four of eight possible cases. The numerical simulations shed light on the remaining cases, and suggest that the impact on w_1 and r_1 are opposite, so that if w_1 increases, r_1 declines, and vice versa.

In order to examine how productivity effects affect the trade-wage link, we have used factor shares k_1 etc. instead of the original factor variables K_1, L_1 etc. While the impact of changes in k_1 on production is ambiguous, an increase in K_1 or L_1 will always increase production. In case we are interested in these direct effects, e.g. for considering growth through factor accumulation, they may be

derived using a method similar to the one described above. Such results are shown in Table A2.

Table A2: The impact of factor endowment changes on factor prices when country 1 is specialised in sector A: Alternative calculations with factor stock variables
Shaded areas: International effects.
Signs in brackets: Suggested by simulation, not confirmed analytically

	$\alpha<\beta$ (sector A is L-intensive)				$\alpha>\beta$ (sector A is K-intensive)			
	K_1	L_1	K_2	L_2	K_1	L_1	K_2	L_2
w_1	(+)	÷	+	÷	(+)	÷	÷	+
r_1	÷	(+)	+	÷	÷	(+)	÷	+
w_1/r_1	+	÷	0	0	+	÷	0	0
w_2	÷	÷	+	÷	+	+	+	÷
r_2	+	+	÷	+	÷	÷	÷	+
w_2/r_2	÷	÷	+	÷	+	+	+	÷
p_A	÷	÷	+	÷	÷	÷	÷	+

Using factor stock variables, it is more difficult to see how the trade-wage link is affected by productivity.

A2. Welfare results

In the following, we provide analytical support for some main conclusions regarding welfare in the case when country 1 is specialised in good A and country 2 is diversified. As in section A1, the results only apply for factor endowment combinations that are consistent with this outcome.

Welfare in autarky is given by (12a), with appropriate country subscripts. For the situation with free trade, consumption levels in country *i* (*i*=1,2) will be $A_i=ap_A^{-1}Y_i$ and $B_i=(1-a)Y_i$. Inserting this into the utility function (8), utility can be expressed as $U_i=\Delta Y_i p_A^{-a}$, where $\Delta=a^a(1-a)^{(1-a)}$ and income $Y_i=r_iK_i+w_iL_i$. We shall consider welfare from two angles; first the relative welfare gains from trade, and second absolute welfare levels. The results are summarised in Table 2 in the main text.

A2.1. The welfare gains from trade

Comparing welfare in autarky versus free trade for the specialised country 1, we obtain after some manipulation:

$$\text{(A11)}\quad \frac{U_1}{U_{1-autarky}}=\left(\frac{\gamma}{\alpha}\right)^{\gamma}\left(\frac{\theta}{1-\alpha}\right)^{\theta}p^{*\,1-a}_{B}$$

where p^*_B is the "shadow price" that would have prevailed if country 1 had produced good B at its actual factor prices. Given that $p_B=1$ and country 1 does not produce B, we know that $p^*_B>1$. In order to check whether the remaining part of the expression is larger or smaller than one, write (observe that $\theta=1-\gamma$)

$$\text{(A12)}\quad \left(\frac{\gamma}{\alpha}\right)^{\gamma}\left(\frac{\theta}{1-\alpha}\right)^{\theta}-1=\frac{\gamma^{\gamma}(1-\gamma)^{(1-\gamma)}-\alpha^{\gamma}(1-\alpha)^{(1-\gamma)}}{\alpha^{\gamma}(1-\alpha)^{(1-\gamma)}}$$

Now observe that for an expression of the type $x^y(1-x)^{(1-y)}$, a maximum is obtained for given y when x=y. Even if γ is a function of α, this simplified approach is sufficient to show that when α≠β and consequently α≠γ, the expression (A12) must be positive. Hence (A11) is larger than one, so country 1 gains from trade.

In order to check how the welfare gain depends on country 1's factor composition, re-write (A11) as

$$\text{(A13)}\quad W_1 = \frac{U_1}{U_{1-autarky}} = \gamma^{\gamma}\theta^{\theta} Z_A^{\ a} Z_B^{\ 1-a} p_A^{\ 1-a} \left(\frac{k_1}{1-k_1}\right)^{(1-a)(\alpha-\beta)}$$

Differentiating with respect to k_1, we obtain

(A14)

$$\frac{\partial W_1}{\partial k_1} = \gamma^{\gamma}\theta^{\theta} Z_A^{\ a} Z_B^{\ 1-a} p_A^{\ -a} \left(\frac{k_1}{1-k_1}\right)^{(1-a)(\alpha-\beta)} (1-a)\left\{\frac{\partial p_A}{\partial k_1} + p_A\ (\alpha-\beta)\frac{1}{k_1(1-k_1)}\right\}$$

From Table 1 in the main text we know that if $k_1<\alpha$ and $\alpha<\beta$, the derivative with respect to p_A is negative. Hence if $k_1<\alpha<\beta$, (A14) is negative so the relative welfare gain from integration is lower, the higher is k_1. However, if $k_1>\alpha$ and $\alpha<\beta$, the derivative with respect to p_A is positive, and in this case, we cannot determine the sign of (A14) unambiguously without an explicit solution.

Correspondingly, if sector A is K-intensive and $k_1>\alpha>\beta$, (A14) will be positive, so the relative welfare gain is larger, the higher is k_1. But if $k_1<\alpha$ and $\alpha>\beta$, the derivative with respect to p_A is negative, and the sign of (A14) is ambiguous.

These results show that *specialised countries with a more extreme factor composition obtain relatively larger gains from trade.*

From (A13), it is also straightforward to derive the impact on welfare of changes in R_1, R_2 and k_2, given that this impact only goes through p_A (see results in Table 1 in the main text). *For the specialised country 1, a size increase reduces the relative welfare gains from trade, while a size increase in country 2 (its trade partner) is good*. The result for R_2 also shows generally that country 1 gains from trade, since if R_2=0, welfare "with trade" must be equal to the autarky level. Furthermore, *the welfare gain is larger, the more different are the K/L ratios*: If good A is L-intensive, an increase in k_2 is good for welfare in country 1, and if good A is K-intensive, the opposite is true.

Turning to the diversified country 2, its welfare in autarky can be expressed as (using 12)

$$\text{(A15)}\quad U_{2-autarky} = \Delta R_2 Z_A^{\ -a} Z_B^{\ a-1} k_2^{\gamma}\left(1-k_2\right)^{\theta} \gamma^{-\gamma}\theta^{-\theta}$$

With free trade, $U_2=\Delta Y_2 p_A^{-a}$ and income is equal to $Y_2 = R_2 (k_2 r_2 + (1-k_2) w_2)$. Using $p_B = 1 = Z_B r_2^{\beta} w_2^{1-\beta}$ (unit cost for good B) as well as the Stolper-Samuelson relationship (4), welfare can be expressed as

$$(A16)\quad U_2 = \Delta Z_A^{-a} Z_B^{a-1} R_2 \left[\frac{w_2}{r_2}\right]^{\gamma-1} \left\{k_2 + (1-k_2)\frac{w_2}{r_2}\right\}$$

Hence the ratio between free trade welfare and welfare in autarky is equal to

$$(A17)\quad W_2 = \frac{U_2}{U_{2-autarky}} = \gamma^{\gamma}\theta^{\theta} k_2^{-\gamma}(1-k_2)^{-\theta}\left\{k_2\left(\frac{w_2}{r_2}\right)^{\gamma-1} + (1-k_2)\left(\frac{w_2}{r_2}\right)^{\gamma}\right\}$$

We shall examine how R_1, R_2 and k_1 affect W_2, for given k_2. Denoting any of these variables by x, we have

$$(A18)\quad \frac{\partial W_2}{\partial x} = \left(\frac{w_2}{r_2}\right)^{\gamma-2} \gamma^{1+\gamma}\theta^{\theta} k_2^{-\gamma}(1-k_2)^{1-\theta} \frac{\partial \frac{w_2}{r_2}}{\partial x}\left[\frac{w_2}{r_2} - \frac{k_2}{1-k_2}\frac{\theta}{\gamma}\right]$$

The derivatives of w_2/r_2 are known from earlier results; hence to interpret the sign of the expression we need to know the sign of the expression in square brackets, which we denote by ω. For this purpose, observe that the expression $k_2\theta/((1-k_2)\gamma)$ to the right is equal to the factor price ratio in autarky (10). With free trade, this factor price ratio must also apply when $k_1=0$, $k_1=1$ and $k_1=k_2$, so in these three situations, the derivative (A18) equals zero. In order to see what happens between these three points, we can use the earlier results for w_2/r_2:

- In the FPE set, w_2/r_2 equals the autarky price when $k_1=k_2$. Furthermore, w^*/r^* is an increasing function of k_1. So (assuming that A is L-intensive) for $k_{1A}<k_1<\gamma$ we must have $\omega<0$, and for $\gamma<k_1<k_{1B}$ we must have $\omega>0$.
- For $0<k_1<\alpha$, w_2/r_2 falls with k_1 if A is L-intensive, so $\omega<0$.
- For $\alpha<k_1<k_{1A}$, w_2/r_2 increases in k_1 if A is L-intensive, so we must have $\omega<0$ also in this range.
- Similarly, we can use former results to show that for $k_{1B}<k_1<1$, we must have $\omega>0$. [19]

From (A18) it is then clear that the derivatives of w_2/r_2 and W_2 must have the same signs for $k_{1B}<k_1<1$, and opposite signs for $0<k_1<k_{1A}$. Using earlier results from Table 1, changes in k_1 affect welfare in country 2 as shown in Figures 5 and 7 in the main text. Furthermore, we find that country 2's relative welfare gain from integration is falling in its own size (R_2), and increasing with the size of its trade partner (R_1). Hence small countries gain more from trade also with complete specialisation. This result can also be used to show that country 2 gains from trade in absolute terms: If $R_1=0$ then welfare in country 2 must equal its autarky welfare. An increase in R_1 will increase welfare; hence trade is better than autarky.

Applying the utility function (8) on L only, we can also show that for labour in the North, trade causes a welfare loss, not only a reduction in nominal income.

[19] The proof also requires that the function $w_2/r_2=f(k_1)$ is continuous. A specific examination is tedious without an explicit solution. We assume that this is fulfilled. All numerical simulations indicate that this is the case.

A2.2. Welfare levels

Using (A16), it is straightforward to find the signs of $\partial U_2/\partial k_1$ and $\partial U_2/\partial R_1$. For interpreting the signs of the resulting expressions, we also use earlier results from Table 1 in the main text (for w_2/r_2), and Section A2.1 (for ω). Unambiguous results on k_2 are hard to find, and derivatives with respect to R_2 are not interesting since U_2 is an aggregate measure that increases with country size.

The utility level in country 1 can be expressed as:

$$(A19) \quad U_1 = \Delta \alpha^{a-1} R_1 r_1^{1-a} k_1^{1+a(\alpha-1)} (1-k_1)^{a(1-\alpha)}$$

The impact of changes in k_2 and R_2 only occur via r_1, and it is seen directly that the signs of $\partial U_1/\partial k_2$ and $\partial r_1/\partial k_2$ must be similar, as well as the signs of $\partial U_1/\partial R_2$ and $\partial r_1/\partial R_2$. We are not able to obtain unambiguous results on the impact of changes in k1, and – as for country 2 – the welfare impact of size changes is not of interest.

The results derived for $\partial U_2/\partial k_1$, combined with results from the numerical simulations, suggest that if columns (3) and (7) in Tables 1 and 2 apply, welfare in country 2 reaches a maximum for $k_1=\alpha$. Without an explicit solution for factor prices, however, clear results on second-order derivatives are difficult to obtain. For country 1, the simulations suggest that a similar maximum applies. For this, we have not been able to provide clear analytical results. We can show, however, that if $R_2 \to \infty$, welfare in country 1 reaches a maximum for $k_1=\alpha$. In this case, the price p_A is determined by the factor endowments in country 2 and may be considered as fixed. Since U_1 can also be written as $U_1=\Delta F_{1A} p_A^{1-a}$, it is evident that for constant p_A, welfare reaches a maximum when production is maximised. For given R_1, this occurs when $k_1=\alpha$.

A3. Specialisation in both countries

We continue assuming that country 1 is specialised in good A; hence equations (15) and (18) still apply. For country 2, which now specialises in B, with production $F_{2B}=K_2^{\beta} L_2^{1-\beta}$, we have similarly

$$(A20) \quad \frac{w_2}{r_2} = \frac{1-\beta}{\beta} \frac{K_2}{L_2}$$

$$(A21) \quad B_2 = (1-a)F_{2B}$$

In value, exports of A from country 1 must equal exports of B from country 2. This gives (still using B as numeraire, $p_B=1$)

$$(A22) \quad (F_{1A} - A_1)\, p_A = (F_{2B} - B_2)$$

or equivalently, using the results above and the results for A,

$$(A22a) \quad (1-a)\, F_{1A}\, p_A = a\, F_{2B}$$

An implication of this, since $Y_1 = F_{1A}\, p_A$ and $Y_2 = F_{2B}$, is that the income ratio is equal to

$$\text{(A23)} \quad \frac{Y_1}{Y_2} = \frac{a}{1-a}$$

Hence consumers use a certain share of their money on each good, and these money accrue to the respective producing countries. The income share of a country is independent of its size; it depends only on the consumption shares!

Equation (A22) also defines the price ratio between the goods, which is

$$\text{(A24)} \quad p_A = \frac{a}{1-a}\frac{F_{2B}}{F_{1A}}$$

Using the unit cost=price relationships (3), we also obtain e.g.

$$\text{(A25)} \quad w_1 = (1-\alpha)\frac{a}{1-a}F_{2B}\frac{1}{R_1(1-k_1)}$$

and the other factor price levels can be derived similarly. Hence w_1 is positively related to

- the cost share 1-α
- the share of demand for the good produced in country 1
- the size and efficiency of the other country (recall $F_{2B}=Y_2$)
- the share k_1; an increase makes L more scarce,

and it is negatively related to country 1's own size. The larger is demand for your product, the larger is country income, and the share to each factor is determined by the production functions.

Welfare levels in the two countries will be

$$\text{(A26a)} \quad U_1^{**} = a\, F_{1A}^{\,a}\, F_{2B}^{\,1-a}$$
$$\text{(A26b)} \quad U_2^{**} = (1-a)\, F_{1A}^{\,a}\, F_{2B}^{\,1-a}$$

The ratio between welfare in the two countries is therefore equal to the income ratio (A23), or a/(1-a). Hence how the "pie is shared", depends only on consumption shares. On the other hand, the "pie may be enlarged" by means of factor endowment increases in either country. With complete specialisation, it is an advantage to be a small country. If e.g. country 2 becomes very large, however, it will become diversified so these results do no longer apply. As shown in Section A2, however, small is beautiful" also in the case with complete specialisation in only one country.

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